Pareto-improving Immigration Policy in the Presence of the Social Security

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November 3, 2010

Abstract

This paper qualitatively and quantitatively analyzes the welfare effect of accepting immigrants in the presence of a pay-as-you-go social security system. First, it demonstrates that if and only if there are inter-generational government transfers from the young to the old in the sense that the marginal product of labor of a young individual is greater than what he or she receives, including publicly provided private good, accepting immigrants Pareto-improves welfare. Second, the paper shows that if there are inter-generational government transfers in the sense defined above, the government can achieve a path that leads to the golden rule level of capital stock per capita within a finite time in a Pareto-improving way by accepting immigrants and by adjusting the wage tax and the capital income tax. Third, this paper presents how those taxes should be adjusted when immigrants are accepted. Fourth, using the computational overlapping generation model developed by Auerbach and Kotlikoff (1987), I simulate the model economy and calculate years needed to reach the golden rule level of capital stock per capita and the present discounted value (PDV) of the welfare gain obtained by accepting immigrants. The simulation shows that there is a trade-off between years needed to arrive at the golden rule level of capital stock per capita and the PDV of the welfare gain. If the government puts the highest weight on the future cohorts in the Pareto-improvement, the years needed to reach the golden rule is shortest and the PDV of the welfare gain is lowest and vice versa. When the government puts the highest weight on the future cohorts, the economy reaches the golden rule level of capital stock per capita around 170 years. At the new steady state, the capital stock per capita increases by 71 percent and the publicly provided private goods per capita increases by 19 percent. The utility index of the cohort born at the new steady state increases by 2.2 percent and the PDV of the utility increases by 0.035 percent

1 Introduction

Accepting a constant flow of immigrants brings a higher population growth rate to a host country. With a neoclassical production function that exhibits the diminishing marginal product of capital, the standard growth model (Solow 1964) predicts that such a higher population growth rate leads to both lower levels of consumption per capita and income per capita, starting from a dynamically efficient initial steady state. On the other hand, recently there are increasing interests among policy makers in accepting immigrants as a policy response to the negative demographic shocks in the presence of a pay-as-you-go social security system. In addition, in the public finance literature, there is an increasing interest in the effect of immigration on the social welfare in a dynamic general equilibrium model in the presence of a pay-as-you-go social security system. By using the computational overlapping generation model (Auerbach and Kotlikoff model(Auerbach and Kotlikoff(1998)), Storesletten(2000) demonstrated that accepting a particular type of immigrants, (skilled old aged immigrants who will not be able to claim the social security benefit due to the requirement of the minimum duration of the social security tax payment) will increase the social welfare in the presence the retirement of the baby boom generation. On the other hand, Fehr, Jokisch and Kotlikoff (2004) argued that such a welfare gain does not exist. Feldstein (2006) analyzed the effect of immigration in Spain and concluded that immigration does not bring welfare gain.

Given the mixed results in the literature regarding the effect of accepting immigrants on the social welfare, a natural question arises whether accepting immigrants Pareto-improves welfare or not from the theoretical point.

This article analyzes such a question. Firstly, using the overlapping generation model developed by Diamond(1965), I present an easily interpretable necessary and sufficient condition under which accepting immigrants can Pareto-improve welfare. Secondly, I demonstrate that accepting immigrants and implementing a relatively simple compensating tax policy can achieve this Pareto-improvement. Thirdly, I analytically show that the government can design immigration and tax policy that lead the economy to the golden rule level of capital stock per capita within a finite time in a Pareto-improving way. Finally, I quantity the welfare effect of accepting immigrants on the economy in the presence of a pay-as-you-go social security system by using the computation overlapping generation model (Auerbach and Kotlikoff model). I demonstrate that with the parameter values that are similar to the ones in the US economy, the model economy reaches the golden rule level of capital stock per capita from 150 to 200 years in a Pareto-improving way. At the new steady state, the capital stock per capita increases by 70 percent and publicly provided private goods per capita increases by 19 percent. The utility level, measured by the expenditure function, increases by 2.2 percent.

The organization of this paper is as follows. In section 2, I present a brief limited literature review. In section 3, I present a theoretical analysis. In section 4, I present a simulation-based analysis using the computation overlapping generation model. In section 5, I present a conclusion.

2 Literature

Storesletten (2003) is the first paper that analyzed the welfare effect of accepting immigrants in a dynamic computational general equilibrium model. In this paper, he showed that if the government selectively accepts immigrants, then it is possible to soften the negative shocks due to the retirement of the baby boom generation. On the other hand, using a more detailed model, Fehr, Jokisch and Kotlikoff (2004) argued that the welfare gain of accepting immigrants is almost zero or very small if it is positive. Center and Feldstein (2006) analyzed the fiscal effect of accepting immigrants in Spain.

In the literature of the effect of the immigration on the host country, several researchers conducted the cost benefit analysis of accepting immigrants (Huddle(1993), Borjas(1994), Passel (1994). Simon (1984) Akbari(1989), Canova and Ravn (1998), Auerbach and Preopoulos (1999), Storesletten(2003)). These analysis suggests that the inclusion of the social security benefit to the retired immigrants is an important factor to determine whether accepting immigrants is beneficial to the host country. However, whether the social security benefit to the retired immigrants should be included as the cost of accepting immigrants is not clear. Under the pay-as-you-go social security system, the social security benefit of the old at the time t is paid by the young at the time t . If a researcher includes the social security benefit of the retired immigrants as a cost, then the tax paid by the children of the immigrants should be included as a benefit. However, in the pay-as-you-go social security system, the payment of the social security tax and the benefit to the retired are approximately the same. This implies that the inclusion of the social security benefit to the retired immigrants as the cost is not necessary. The analysis of the current paper shows that how the cost and benefit should be calculated when immigrants are accepted in the presence of a pay-as-you-go social security system.

3 Analysis

3.1 The model

The model uses the standard overlapping generation model with a neoclassical production function developed by Diamond (1965). At every period, a continuum of individuals is born. Individuals who are born at period t is denoted as cohort t. Each cohort lives for two periods. When individuals are in the first period, they work and they are called "young". When they are in the second period, they are retired and they are called "old". I assume that immigrants come only when they are young and that the government at the host country prohibits the immigrants to immigrate when they are already old. . Each cohort of the native and the immigrants supplies one unit of labor inelastically when they are young[1]. It is assumed that the preference of the native and the immigrants are the same. Let c_t^y and c_{t+1}^o be the consumption at the young period

[1]In section 3.3, I show that the main results of this paper hold even when the labor supply is elastic. I use the assumption of the inelastic labor supply to simplify the analysis in this section.

and the old period of the cohort t, who are born at the period t. Let g_t^y be the amount of publicly provided private goods per capita for the old that is consumed at period t such as education and government provided health care service for the young. Let g_t^o be the amount of publicly provided private goods per capita for the old at the period t such as medicaid and publicly provided nursing home. Let g_t^{ind} be the amount of age-independent publicly provided private goods per capita such as police service. [2] I assume that the utility function of cohort t is

$$U(c_t^y, c_{t+1}^o, g_t^y, g_{t+1}^o, g_t^{ind}, g_{t+1}^{ind}) = u_y(c_t^y) + v_y(g_t^y, g_t^{ind}) + \frac{1}{1+\rho}[u_o(c_{t+1}^o) + v_o(g_{t+1}^o, g_{t+1}^{ind})] \qquad (1)$$

I assume that $u_y(c_t^y)$, $u_o(c_{t+1}^o)$,$v_y(g_t^y, g_t^{ind})$ and $v_o(g_{t+1}^o, g_{t+1}^{ind})$ are strictly increasing and concave function. I assume additive separability of publicly provided private goods so that the provision of publicly provided private goods does not affect the consumption and saving decision of consumers. This assumption simplifies the analysis since I assume that the government redistributes the welfare gain of accepting immigrants in the form of increased publicly provided private goods to consumers.[3] The assumption of the above utility function also implies that labor supply is inelastic. This assumption also simplifies the analysis. But in the section 3.3, I will show that main result of this paper hold even if the labor supply is elastic. Let s_t be the saving of those who are born or immigrate at the period t. Let w_t and r_t be the pre-tax wage rate and interest rate at period t. Let τ_{rt} and τ_{wt} be the tax rate on interest rate and the wage rate at the period t. Let b_t be the social security benefit that is given to the old at period t. Cohort t maximizes the above utility function subject to the budget constraint. The budget constraint of the cohort t is

$$w_t(1-\tau_{wt}) = c_t^y + s_t \text{ and } b_{t+1} + (1+(1-\tau_{rt})r_t)s_t = c_{t+1}^o \qquad (2)$$

[2]In this paper, I ignore non-rivalry public goods. Note that the presence of non-rivalry public goods will favor immigration since accepting immigration means that the cost of non-rivalry public goods will be shared by more individuals with the same amount of non-rivalry public goods per capita.

[3]There are several ways to redistribute the welfare gain to consumers. I use this method to simplify the analysis. Using other ways for redistributing the welfare gain, the main conclusion does not change.

For the production side, let $F(L_t, K_t)$ be a production function where L_t and K_t are the total capital stock and the total amount of labor used at period t, respectively. I assume that $F(L_t, K_t)$ exhibits constant returns to scale and that both the marginal product of labor and capital are diminishing. Let δ and N_t be the depreciation rate of the capital and the number of the young at the period t. I assume that the standard Inada condition is satisfied:

$$F(x, 0) = 0, \ \lim_{k+\to 0} \frac{\partial F(x,k)}{\partial K} \to \infty, \quad \lim_{k+\to \infty} \frac{\partial F(x,k)}{\partial K} \to 0 \tag{3}$$

The purpose of this paper is to analyze whether accepting immigrants can Pareto-improve welfare or not. In the literature, it is well-known that if the market interest rate is lower than the population growth rate, it is possible to Pareto-improve welfare (dynamic inefficiency). Since this paper's interest is not such a dynamic inefficiency problem, I postulate that at the initial steady state, the market interest rate is higher than the population growth rate (Cass(1972)). Furthermore, for the welfare analysis of immigration, I make the following additional assumptions.

AS1: The economy is at the steady state before the government accepts immigrants.

AS2: The amount of publicly provided private goods, $(g_t^y, g_t^o, g_t^{ind})$ per person is constant over time and $(g_t^y, g_t^o, g_t^{ind}) = (g^y, g^o, g^{ind})$.

AS3: The government uses a pay-as-you-go social security system at the initial steady state.

AS4: Immigrants and the native must be treated equally in the tax system once the immigrants are accepted in the host country.

AS5: For the treatment of immigrants regarding the social security system, the government collects the social security tax from the immigrants in the same way as it collects from the native and it pays the benefit in the same way as to the native

AS6: Immigrants and the native has the same productivity(same efficient unit of labor).

AS7: Immigrants and the native has the same preference.

AS8: The descendant of immigrants assimilates to the native and earns the same income as the native.

AS9: Immigrants and their children stay permanently in the host country.

AS10: The fertility rate of immigrants is equal to or higher than the fertility of the native.

AS11: At the initial steady state, the government does not impose a distorting tax such as capital income tax.

AS1 is standard for the policy analysis where the model involves a dynamic dimension. I use the AS2 to focus on the issue of immigration rather than issues on public expenditure. AS3 comes from the fact that the social security of most countries is the pay-as-you-go system. AS 4 and AS 5 need more discussion. Clearly, if the government can treat the immigrants discriminatory in the tax and pension systems, there is a way to increase the utility of both native and the immigrants. Since the wage rate of the immigrants in their home country is lower than the wage rate in the host country, if the government in the host country can set a high tax rate on incoming immigrants in a way that the net wage rate of the immigrants in the host country is still higher than the wage rate of the immigrants in the home country and if the government redistributes the tax revenue collected from the immigrants to the native, it is possible to Pareto-improve welfare of both the native and the immigrants. AS4 and AS5 delete such an obvious case. In addition, AS4 can be justified from a political reason. Although imposing a high tax rate on the incoming immigrants and redistributing the tax revenue to the native can be Pareto-improving for both the native and the immigrants, such a discriminatory policy might be sometimes illegal from a constitutional point. Also such an discriminatory policy is not feasible when the native does not want to see the immigrants treated in a discriminatory way. Thus, in the following analysis, I assume that in the tax and social security policy, the government must treat the immigrants and the native in the same way. Even under such a constraint, I will show that it is possible to Pareto-improve welfare by accepting immigrants if a certain condition is met. The AS6 and AS7 are not critical for proposition 1. AS8, AS9 and AS10 are used to simplify the analysis. I use AS11 because of our interest in the welfare effect of immigration. If there is already distorting tax at the initial steady state in the economy, clearly the government can Pareto-improve the initial steady state by correcting those taxes. Because the labor supply is inelastic, the wage tax is optimal and the use of capital income tax is sub-optimal. Since the focus of this paper is the first order effect of immigration, not the tax reform, I use the AS11. Of course, in reality, the governments of the most developed countries impose capital income taxes. Thus, in the section 3.3, I show that the main conclusion does not change even in the presence of capital income taxation. [4]

Now consider the government budget constraint at period t. At this point, consider the government budget constraint with the capital income tax although it is assumed that the capital income tax is not used at the initial steady state. The government budget constraint at period t is

$$\tau_{wt+1}w_{t+1}N_{t+1}+\tau_{rt+1}r_{t+1}s_tN_t-N_t(b_t+g^o+g^{ind})-N_{t+1}(g^y+g^{ind})+(1+r_t)N_ta_t-N_{t+1}a_{t+1}=0. \tag{4}$$

where a_t is the government saving (debt) balance per capita at period t. b_t is the social security benefit at the period t.

Then, w_{t+1} and r_{t+1} are determined as follows:

$$w_{t+1}=\frac{\partial F(N_{t+1},(s_t+a_t)N_t)}{\partial L} \text{ and } r_{t+1}=\frac{\partial F(N_{t+1},(s_t+a_t)N_t)}{\partial K}-\delta \tag{5}$$

From the individual budget constraint, $w_{t+1}(1-\tau_{wt+1})=c^y_{t+1}+s_{t+1}$ and $c^o_{t+1}=(1+r_{t+1}(1-\tau_{rt+1}))$. Solving for τ_{wt+1} and τ_{rt+1} and substituting them into the government budget constraint, we have the following resource constraint:

$$F(N_{t+1},(s_t+a_t)N_t)+(1-\delta)(s_t+a_t)N_t=\{c^y_{t+1}+s_{t+1}+a_{t+1}+g^y+g^{ind}\}N_{t+1}+N_t\{c^o_{t+1}+g^o+g^{ind}\} \tag{6}$$

[4] Technically I assume that the labor supply is inelastic, because non-uniqueness of the second best taxes. Non-uniqueness of the second best tax implies that the " only if" part of the proposition 1 does not hold when the labor supply is elastic.

Now, let nat_t be the number of the young native at the period t and imi_t be the number of young immigrant at the period t. Let N_t be the population of the young at the period t and α_t be the ratio of immigrant to the native. Let the fertility rate of the native and the immigrant be π_n and π_m. Then, N_t can be written as follows:

$$N_t = imi_t + nat_t \ , \ imi_t = \alpha_t \times nat_t$$

$$\text{and } nat_t = (1 + \pi_m) \times imi_{t-1} + (1 + \pi_n) \times nat_{t-1}. \tag{7}$$

The immigration policy is expressed in terms of α_t. For example, one time acceptance of immigration means that $\alpha_0 = 0$, $\alpha_1 = \alpha$ and $\alpha_t = 0$ for $t = 2, 3, ...$ Permanently accepting immigrants means that $\alpha_0 = 0$ and $\alpha_t = \alpha$ for all $t = 1, 2, , , ,$. Now, for convenience, calculate the ratio of immigrant to the population of the young, which is calculated $imi_t/(imi_t + nat_t)$ and define as ϕ_t:

$$\frac{imi_t}{imi_t + nat_t} = \frac{\alpha_t}{\alpha_t + 1} \equiv \phi_t(\alpha_t) \tag{8}$$

Note that ϕ_t is an increasing function of α_t and it is equal to zero when $\alpha_t = 0$. Also, $d\phi_t/d\alpha_t > 0$ and $d^2\phi_t/d\alpha_t^2 < 0$. From the definition of N_t, The (immigrant included) population growth rate is

$$\frac{N_{t+1}}{N_t} - 1 = (1 + \alpha_{t+1}) \times \{(1 + \pi_m)\phi_t + (1 + \pi_n)(1 - \phi_t)\} - 1 \tag{9}$$

Thus, we can write the population growth rate, $N_{t+1}/N_t - 1$, as function of α_{t+1} and ϕ_t, $\Omega($ $\alpha_{t+1}, \phi_t)$ and it is denoted as $\Omega(\alpha_{t+1}\,\phi_t)$. $\Omega(\alpha_{t+1}\,\phi_t)$ is the immigration included population growth rate at period t+1 when the ratio of the immigrants to the young native at period $t+1$ is α_{t+1} and the ratio of immigrant to the population of the young at period t is ϕ_t. By calculating the derivative of Ω with respect to α_{t+1} and ϕ_t, we have

$$\frac{\partial\Omega}{\partial\alpha_{t+1}} = \{(1 + \pi_m)\phi_t + (1 + \pi_n)(1 - \phi_t)\} > 0 \ , \ \frac{\partial\Omega}{\partial\phi_t} = (1 + \alpha_{t+1})(\pi_m - \pi_n) > 0. \tag{10}$$

$\{(1 + \pi_m)\phi_t + (1 + \pi_n)(1 - \phi_t)\}$ is the direct effect of immigration on population growth rate. Accepting immigrants means that it brings more working young. Thus it increases the population growth rate. $(1+\alpha_{t+1})(\pi_m - \pi_n)$ is the indirect effect of accepting immigrants on population growth rate. Because of the assumption that the fertility rate of immigrants is higher or equal to the one of the native, having higher a ratio of immigrants implies a higher population growth rate.

Note that we can divide the both sides of the resource constraint by $N_{t\cdot}$. Then, the resource constraint can be rewritten as

$$F(1+\Omega(\alpha_{t+1},\phi_t), s_t+a_t)+(1-\delta)(s_t+a_t) \geq c_t^o+g^o+g^{ind}+(1+\Omega(\alpha_{t+1},\phi_t))(c_{t+1}^y+s_{t+1}+g^y+g^{ind}) \ . \tag{11}$$

Now consider the individual intertemporal optimization problem. The individual intertemporal first order condition is

$$(1+\rho)\frac{u_y'(c_t^y)}{u_o'(c_{t+1}^o)} = 1 + r_{t+1}(1-\tau_{rt+1}) \tag{12}$$

Also consumption at the old is

$$c_{t+1}^o = s_t \times (1 + r_{t+1}(1-\tau_{rt+1})) + b_t \tag{13}$$

This implies that the following relationship must hold among c_t^y and c_{t+1}^o and s_t:

$$s_t \times (1+\rho)u_y'(c_t^y) - (c_{t+1}^o - b_t)u_o'(c_{t+1}^o) = 0. \tag{14}$$

Before analyzing the effect of accepting immigrants, it would be useful to characterize the initial steady state. Let w^* and r^* be the pre-tax wage rate and the interest rate at the initial steady state, respectively. Let s^* and N_0 be the individual saving and the number of old people at the period 0. Let τ_w^* be the wage tax rate at the initial steady state. Let b^* be the social security benefit at the initial steady state.

The steady-state economy with zero capital income tax is characterized as follows:

$$s^* = \arg\max_s u_y(w^*(1-t_w^*)-s) + v_y(g^y, g^{ind}) + \frac{1}{1+\rho}[u_o((1+r^*)s+b) + v_o(g^o, g^{ind})] \quad (15)$$

$$w^* = \frac{\partial F}{\partial L}((1+\pi_n)N_0, N_0 s^*)$$

$$r^* + \delta = \frac{\partial F}{\partial K}((1+\pi_n)N_0, N_0 s^*)$$

$$\tau_w^* w^*(1+\pi_n)N_0 - N_0(b^* + g^o) - N_0(1+\pi_n)g^y = 0$$

where t_w^*, g^y, g^o,g^{ind}, b^* are given.

Also define the steady state level of the consumption and the utility with zero capital income tax as follows:

$$c^{y*} = w^*(1-t_w^*) - s^*,\ c^{o*} = (1+r^*)s^* + b^* \quad (16)$$

$$u^* = u_y(c^{y*}) + v_y(g^y, g^{ind}) + \frac{1}{1+\rho}[u_o(c^{o*}) + v_y(g^o, g^{ind})]. \quad (17)$$

3.2 Welfare Effect of Accepting Immigrants

In this sub-subsection, I examine whether permanently continuing to accept immigrants will Pareto-improve welfare or not.

Before analyzing the welfare effect of accepting immigrants, first note that the above steady-state with zero capital income tax is efficient since labor supply is inelastic.

Observation 1 Given the constraint that the government does not accept the immigrant, the above steady-state economy with zero capital income tax is Pareto-efficient.

Proof. See appendix.

Now, I consider a case where the government permanently accepts immigrants with a constant ratio. Permanently accepts immigrants with a constant ratio means that $\alpha_0 = 0$ and $\alpha_t = \alpha$ for $t = 1, 2.3, ...$ Let $\phi_t(\alpha)$ be $\phi(\alpha)$. Then, the calculate the $d\Omega/d\alpha$ and $d^2\Omega/d\alpha^2$:

$$\frac{d\Omega(\alpha,\phi(\alpha))}{d\alpha} = 1 + \pi_m > 0 \text{ and } \frac{d^2\Omega(\alpha,\phi(\alpha))}{d\alpha^2} = 0 \quad (18)$$

Now, consider the following constrained maximization problem:

$$\text{Main programming problem} \tag{(MPP)}$$

$$V(\alpha) = \max_{\{c_t^y, c_t^o, s_t,\ a_t | t=1,2,..\}} \frac{1}{1+\rho}[u_o(c_1^o) + v_o(g_o, g^{ind})] \tag{19}$$

$$\text{s.t. } u(c_t^y) + v_y(g^y, g^{ind}) + \frac{1}{1+\rho}[u^o(c_{t+1}^o) + v_o(g^o, g^{ind})] \geq u^* \quad \text{for } t = 1, 2, ... \tag{20}$$

$$F(1+\Omega(\alpha,\phi(0)), s^*) + (1-\delta)s^* \geq (c_1^o + g^o + g^{ind}) + (1+\Omega(\alpha,\phi(0))(c_1^y + s_1 + a_1 + g^y + g^{ind}) \tag{21}$$

$$F(1+\Omega(\alpha,\phi(\alpha)), s_t + a_t) + (1-\delta)s_t \geq \tag{22}$$

$$(c_t^o + g^o + g^{ind}) + (1+\Omega(\alpha,\phi(\alpha)) \times (c_{t+1}^y + s_{t+1} + g^y + g^{ind}) \text{ for } t = 1, 2, .. \tag{23}$$

The above programming problem deserves several comments. First, $V(\alpha)$ is the utility of the cohort 0 at the period 1 when the government accepts immigrant permanently with a constant ratio α. Second, the first constraint is related with Pareto-improvement and it requires that the all cohort except cohort 0 need to have at least as the same utility that they would have at the initial steady state. Second, (21) and (23) are the resource constraints. Instead of the government budget constraint, we use the resource constraint since both are equivalent. Third, in those resource constraints, the tax and social security are not defined. But the once the consumption and saving level are defined, the tax and social security benefit is calculated implicitly. To demonstrate, suppose that c_t^y, c_t^o, s_t and a_t are determined. Then, w_t, r_t, t_{wt}, t_{rt} and b_t are calculated from the following equations:

$$w_t : w_t = \frac{\partial F}{\partial L}(N_t, N_{t-1}(a_{t-1} + s_{t-1})) \tag{24}$$

$$r_t : r_t = \frac{\partial F}{\partial K}(N_t, N_{t-1}(a_{t-1} + s_{t-1})) - \delta \tag{25}$$

$$\tau_{wt} : w_t(1-\tau_{wt}) = c_t^y + s_t \tag{26}$$

$$\tau_{rt} : (1+\rho)\frac{u_y'(c_t^y)}{u_o'(c_{t+1}^o)} = 1 + r_{t+1}(1-\tau_{rt+1}) \tag{27}$$

$$b_t : s_t \times (1+\rho)u_y'(c_t^y) - (c_{t+1}^o - b_t)u_o'(c_{t+1}^o) = 0 \tag{28}$$

Fourth, note that the objective function of (MPP) is strictly concave and the constraint set is convex with respect to control variables. Furthermore,the resource constraints (21) and (23) are strictly concave with respect to α. This implies that we have the following observation:

Observation 2 $V(\alpha)$ is strictly concave.

Proof See appendix 3.

Let γ_t be the Lagrangian multiplier of (20) and let λ_t be the Lagrangian multiplier of (23). Then, the Lagrangian function is

$$
\begin{aligned}
L &= \frac{1}{1+\rho}[u_o(c_1^o) + v_o(g^o, g^{ind})] \\
&+ \sum_{t=1}^{\infty} \gamma_t \{U(c_t^y, c_{t+1}^o, g^y, g^o, g^{ind}, g^{ind}) - u^*\} \\
&+ \lambda_1 \{F(1+\Omega(\alpha,0), s^*) + (1-\delta)s^* - (c_1^o + g^o + g^{ind}) - (1+\Omega(\alpha,0)) \times (c_1^y + s_1 + a_1 + g^y + g^{ind})\ \} \\
&+ \sum_{t=2}^{\infty} \lambda_t \{F(1+\Omega(\alpha,\phi(\alpha)), s_{t-1} + a_{t-1}) + (1-\delta)(s_{t-1} + a_{t-1}) \\
&- (c_t^o + g^o + g^{ind}) - (1+\Omega(\alpha,\phi(\alpha))) \times (c_t^y + s_t + a_t + g^y + g^{ind})\ \}
\end{aligned}
\tag{29}
$$

Before calculating $V'(\alpha)$, it would be useful to characterize the solution of the above programming problem. Let γ_i^* be the Lagrangian multiplier of the utility constraint at period t when $\alpha = 0$. Let λ_t^* the Lagrangian multiplier of the resource constraint of period t when $\alpha = 0$. Then, we have the following observation.

Observation 3

When $\alpha = 0$ the solution of MPP is

$$c_t^y = c^{y*},\ c_0^o = c_t^o = c^{o*},\ s_t = s^*,\ a_t = 0 \text{ for } t = 1, 2, ... \tag{30}$$

$$\lambda_1^* = \frac{1}{1+\rho} u_o'(c_1^o, g_o, q) \text{ and } \lambda_{t+1}^* = \frac{1+\pi_n}{1+r^*}\lambda_t^* \tag{31}$$

$$\gamma_t^* = \frac{1}{u_y'(c^{y*})}\lambda_t^*(1+\pi_n) \text{ and for } t = 1, 2, ... \tag{32}$$

Proof See appendix 4.

Observation 3 comes from Observation 1. Since the initial steady-state economy with zero capital income tax is efficient, we can calculate the Lagrangian multiplier of the programming problem when immigrants are not accepted.

Now, suppose that the government accepts immigrant permanently with a constant proportion. Whether such acceptance of immigrants will Pareto-improve the welfare or not can be analyzed by calculating $V'(\alpha)$ and evaluate at $\alpha = 0$. From the envelope theorem,

$$\begin{aligned} \left.\frac{dV}{d\alpha}\right|_{\alpha=0} &= \left.\frac{\partial L}{\partial \alpha}\right|_{\alpha=0} \\ &= \frac{\partial \Omega(0,0)}{\partial \alpha}\lambda_1\{\frac{\partial F(1+\pi_n, s^*)}{\partial L} - c_1^y - s_1 - g^y - a_t - g^{ind}\} \\ &+ \frac{d\Omega(0,0)}{d\alpha}\sum_{t=2}^{\infty}\lambda_t\{\frac{\partial F(1+\pi_n, s_t)}{\partial L} - c_t^y - s_t - g^y - a_t - g^{ind}\}. \end{aligned} \tag{33}$$

Note that at $\alpha = 0$, $\lambda_t = \lambda_t^*$, $d\Omega/d\alpha = 1 + \pi_m$ and $a_t = 0$. From (30), $c_t^y = c^{y*}$, $s_t = s^*$. Using the homogeneity of production function, we have

$$\left.\frac{dV(\alpha)}{d\alpha}\right|_{\alpha=0} = \left\{(1+\pi_n)\lambda_1^* + (1+\pi_m)\sum_{t=2}^{\infty}\lambda_t^*\right\} \times \left\{\frac{\partial F(N_0(1+\pi_n), s^* N_0)}{\partial L} - c_y^* - s^* - g^y - g^{ind}\right\} \tag{34}$$

The first bracket is positive since the Lagrangian multiplier of the resource constraint is positive. As for the second bracket, the first term is the marginal product of labor and it is what the immigrants brings to the economy. It is also the pre-tax wage income of the young at the initial steady state. $c_y^* + s^*$ is the after tax income and it is also the private resource used by the young at the initial steady state. $g_y + g^{ind}$ are the private goods that are provided by the government for the young. Therefore, $c_y^* + s^* + g^y + g^{ind}$ is the resource allocated to the young while they are young at the initial steady state. This implies that the inside of the bracket is the pre-tax wage minus the resource allocated to the young at the initial steady state. Therefore, the bracket is the amount of an intergenerational transfer from the young to the old. For example, if the inside of the second bracket is equal to zero, it implies that what the young earns is fully allocated to the young. If the inside of the bracket is positive, it implies that the resource allocated to the young is lower than what the young earns. Furthermore, since $V(\alpha)$ is strictly concave, it is not possible to increase $V(\alpha)$ by choosing any $\alpha \geq 0$ if $V'(\alpha) \leq 0$. Thus, we have the following proposition.

Proposition1. (MPL condition) *If there is an intergenerational transfer from the young to the old at the initial steady state in the sense that the marginal product of labor of the young is greater than the resource allocated to the young(private consumption, saving plus publicly provided private goods of the young), then accepting immigrants is Pareto-improving for all generations. If there is no intergenerational transfer at the initial steady state, then it is not possible to Pareto-improve welfare by accepting immigrant.*

Graphically, Proposition 1 can be explained as follows. To illustrate, consider a simple case where the depreciation rate is 100% ($\delta = 1$) and the fertility rate of the native and immigrant are the same. At period 0, the following resource constraint must hold

$$F((1+\pi_n)N_0, N_0 s^*) - N_0(c^{o*} + g^o + g^{ind}) - N_0(1+\pi_n)(c^{y*} + s^* + g^y + g^{ind}) = 0 \tag{35}$$

By dividing both side by N_0, we have

$$F(1+\pi_n, s^*) - (c^{o*} + g^o + g^{ind}) - (1+\pi_n)(c^{y*} + s^* + g^y + g^{ind}) = 0 \tag{36}$$

Note that $F(1+\pi_n, s^*)$ is the output per old when the population growth rate is equal to π_n. Now consider the graph of (y, x) where $y = F(x, s^*)$. This is the graph that show the relationship between one plus the population growth rate and the output per old. At $x = 1+\pi_n$, y represents the output per old at the initial steady state. Next, draw the line of (y, x) where y is defined as $y = (c^{y*} + s^* + g^y + g^{ind})x$. Note that $(c^{y*} + s^* + g^y + g^{ind})x = \frac{(c^{y*}+s^*+g^y+g^{ind})xN_0}{N_0}$. Thus, $y = (c^{y*} + s^* + g^y + g^{ind})x$ represents the total resource used for the young divided the number of old when $x - 1$ is the population growth rate. This line passes the origin and the slope is $c^{y*} + s^* + g^y + g^{ind}$. At $x = 1+\pi_n$, the vertical distance of this line represents the total amount of resource used for young divided by the number of old when the population growth rate is equal to π_n. Thus, the difference between $y = F(x, s^*)$ and $y = (c^{y*} + s^* + g^y + g^{ind})x$ at $x = 1+\pi_n$ represents the amount of resource used for one representative old.

Now suppose that a social planner increases the population growth rate by accepting immigrants permanently. This implies that x moves to right from $x = 1 + \pi_n$. If the slope of $y = F(x, s^*)$ at the $x = 1 + \pi_n$ is greater than $c^{y*} + s^* + g^y + g^{ind}$, then the social planner can maintain the same resource per young (private consumption, saving and governed provided private goods and age independent public goods) and increase the resource per old. Clearly this is the Pareto-improvement. (See figure 1). Note that when the government accepts immigrants there is a surplus that is equal to $\partial F(1 + \pi_n, s^*)/\partial L - (c^{y*} + s^* + g^y + g^{ind})$ at every period. Also note that $\sum \lambda_t^*$ is the present discounted value of increasing the resource for all periods. Therefore, (34) presents the present discounted value of the surplus of all periods obtained by increasing the population growth rate.

Because the marginal product of labor of the young is the pre-tax wage of the young at the initial steady state and $c^{y*} + s^*$ is the after-tax income of the young by the definition. Thus, $\frac{\partial F(N_0(1+\pi_n), s^* N_0)}{\partial L} - c^{y*} - s^*$ is the amount of tax paid by the young. This implies that if the amount of tax paid by the young is greater than the goods provided by the government to the young at the initial steady state, accepting immigrants is Pareto-improving:

Corollary (Tax condition) *Accepting immigrant Pareto-improve welfare if and only if the amount of the tax that the immigrant pays is greater than the publicly provided private good that the immigrant receive while they are young.*

This corollary is useful when we need to conduct the cost-benefit analysis of accepting immigrants. [5] When researchers calculate the cost and benefit of accepting immigrants, the inclusion of the social benefits that the immigrant receive is critical to determine the cost of accepting immigrants. Some researchers argues that the social security tax that the immigrants pays and the social security benefit that immigrants receives should included for the cost-benefit calculation. (First view). However, other researchers argue that if the social security benefit that immigrant

[5]Note that the tax condition does not change even in the presence of public goods since an increase of the immigrants does not affect the consumption of public goods by the definition of public goods.

receives is included, then the social security tax that children of the immigrant pays should be included since the social security benefits of the old is paid by the children of the immigrants through the social security tax. But if the social security is roughly a pay-as-you-go system, then the social security benefit of the immigrants receives and the social security tax that the children of the immigrants pay is roughly balanced. This implies that the social security benefit should not be included and only the social security tax that the immigrants pays should be included(Second view).

The above tax condition indicates that the second view is correct. The welfare effect of immigrant depends on what the tax new immigrant pays minus what the immigrant receives as publicly provided private goods when they are young. It shows that the social security benefit that the immigrant receives when the are old does not matter.

At this point, it would be useful to discuss the limit of the proposition 1. "only if" part of the Proposition 1 clearly depends on the assumption that $V(\alpha)$ is concave. Concavity of $V(\alpha)$ comes from the assumption that labor supply is inelastic. When labor supply is elastic, the indirect utility function generated by the optimal taxes is not concave with respect to an exogenous parameter in general due to the nature of the second best problem. Thus, "only if" part does not hold when labor supply is elastic.

On the other hand, "if" part of the Proposition 1 is quite robust. In the next subsection, I shows that even if labor supply is elastic and even if the capital income tax and labor income taxes at the initial steady state are not optimal, accepting immigrant Pareto-improves welfare if MPL condition is satisfied. In addition, the analysis in the next subsection show that if MPL condition is satisfied, then the government can design immigration policy so that the capital stock of the economy leads to the golden rule level in a finite time in a Pareto-improving way.

3.3 Elastic labor Supply and the Presence of Distortionary Taxes

The previous subsection has shown that it is Pareto-improving to accept immigrants when there are intergenerational government transfers between young agents and old agents in the sense that the marginal product of labor of the young is greater than what is allocated to the young. However, the analysis does not show how such a Pareto-improvement is achieved. When immigrants come, the pre-tax wage rate will decrease and the pre-tax interest rate will increase due to the assumption of diminishing marginal product of labor. To Pareto-improve welfare all generations, adjustment of taxes is needed.

Also, in the previous subsection, I have assumed that the labor supply is inelastic and that there is no capital income tax at the initial steady state.

This subsection shows that if the MPL condition holds, accepting immigrants with a relatively simple tax adjustment Pareto-improves welfare and that this is true even if the labor supply is elastic and there are (optimal or non-optimal) taxes (including capital income tax) at the initial steady state.

The tax adjustment used when immigrants are accepted is that the government adjusts the wage tax rate and the capital income tax rate so that after tax wage rate and after tax interest rate become the same as at the initial steady state. With this relatively simple tax adjustment, government can Pareto-improve welfare by accepting immigrants when MPL condition is satisfied.

For the analysis, therefore, assume that labor supply is elastic and that the government uses a capital income tax and the wage tax at the initial steady state. Assume that those taxes can be non-optimal. Second, motivated by Proposition 1, assume that MPL condition is satisfied at the initial steady state:

$$\frac{\partial F(N_0(1+\pi_n), N_0 s^*)}{\partial L} > c^{y*} + s^* + g^y + g^{ind} \quad . \tag{37}$$

The result in the previous section shows that, starting from zero immigrants, a marginal increase of immigrants Pareto-improve welfare if MPL condition is satisfied. But that result does not implies unlimited acceptance of immigrants always Pareto-improve welfare. I assume the government chooses α so that MPL condition is not violated strictly.

$$\frac{\partial F(N_0(1+\Omega(\alpha,\phi(\alpha))),N_0 s^*)}{\partial L} \geq c^{y*} + s^* + g^y + g^{ind} \tag{38}$$

When the government accepts immigrant and adjusts the wage tax rate and capital income tax rate so that after tax rate and after tax interest rate are the same as at the initials steady state, the government can accumulate the government saving as I prove below. This implies that it is possible that at some point, the marginal product of capital (MPK) becomes equal to the golden level of capital stock per capita. But when MPK becomes equal to the golden rule level, it is clearly better to use the all government surplus to increase the supply of publicly provided private goods rather than to increase the government saving balance. Thus, I assume that as long as MPK is higher than the golden rule level, the government uses some of the government budget surplus to increase the government saving balance and uses the rest to increase the supply of publicly provided private goods. When MPK reaches the golden rule level, the government uses all surplus to increase the publicly provided private goods. This implies that

$$\frac{\partial F}{\partial K} \geq \delta + \Omega(\alpha,\phi(\alpha)) \tag{39}$$

Now let $\partial F/\partial L$ be w_t and let τ_{wt} be the wage tax rate at period 1 when the immigrants are accepted. Let τ_{rt} be the tax rate on the interest income at period t. Let τ_r^* be the capital income tax rate at the initial steady state. To achieve Pareto-improvement when the government accepts immigrants, the government sets the wage tax rate and the interest tax rate at period t in the following way:

$$w_t(1-\tau_{wt}) = w^*(1-\tau_w^*) \text{ and } r_t(1-\tau_{rt}) = r^*(1-\tau_r^*) \tag{40}$$

In other words, the government sets the tax rates so that after tax wage and interest rate after the acceptance of immigrants are equal to after tax wage rate and interest rate at the initial steady state. Also assume that the government gives the same social security benefit as at the initial steady state. This implies that $b_t = b$ for $t = 1, 2, 3...$ When the government sets taxes and social security benefit in this way, saving behavior and labor supply behavior do not change because the consumers have the same after tax wage and interest rate as at the initial steady state. If the government provides at least as the same level of publicly provided private goods, the levels of the utility of all cohorts are at least as the same as at the initial steady state. The question that we examine is whether such taxes are feasible from the point of the government budget constraint. To check the feasibility of such taxes, consider the net government budget surplus at the period 1, SP_1 :

$$SP_1 = w_1 \tau_{w1} N_1 + r_1 \tau_{r1} s^* N_0 - N_0 \times (b + g^o + g^{ind}) - N_1 \times (g^y + g^{ind}) \tag{41}$$

Note that from

$$\tau_{w1} = 1 - \frac{w^*(1-\tau_w^*)}{w_1} \text{ , } \tau_{r1} = 1 - \frac{r^*(1-\tau_r^*)}{r_1} \text{ and } N_1 = N_0(1 + \Omega(\alpha, 0)) \tag{42}$$

By substituting τ_{w1} and τ_{r1} into SP_1 and using homogeneity of production function, we have (see appendix 5)

$$= N_0 \int_{1+\pi_n}^{1+\Omega(\alpha,0)} [\frac{\partial F(zN_0, , N_0 s^*)}{\partial L} - c^{y*} - s^* - g^y - g^{ind}] dz \tag{43}$$

Note that $\Omega(\alpha, \phi(\alpha)) > \Omega(\alpha, 0) > \pi_n$ for $\alpha > 0$. From the MPL condition and the assumption on the choice of α,(38), the inside of the integration is positive for $z \in [1 + \pi_n,\ 1 + \Omega(\alpha, 0)]$.

This means that this tax plan is feasible in the period 1. The government can use some of the surplus of the budget to increase the supply of publicly provided private goods and accumulate the rest as the government saving. Let a_1 be the balance of the government savings per young population at the end of period 1. How about the net government surplus in period 2, SP_2 ?

Note that the consumers who is born at the period 1 will save as the consumers at the initial steady state economy since the consumers born in the period 1 face the same after-tax wage and interest rate as the consumers at the initial steady state under the proposed tax policy. This implies that $s_1 = s^*$ for both consumers born at the period 1 and immigrants who come at the period 1. Therefore, the total capital stock and labor force at the period 2 are $N_1(s^* + a_1)$ and $N_1(1+\Omega(\alpha,\phi(\alpha)))$, respectively. SP_2 becomes

$$SP_2 = w_2\tau_{\tau w2}N_2 + r_2\tau_{r2}s^*N_1 - N_1 \times (b + g^o + g^{ind}) - N_2 \times (g^y + g^{ind}) + (1+r_2)a_1N_1. \quad (44)$$

Note that the pre-tax wage at period 2, w_2, and the pre-tax interest rate at period 2, r_2, are equal to

$$w_2 \equiv \frac{\partial F(N_1(1+\Omega(\alpha,\phi(\alpha))), N_1(s^*+a_1))}{\partial L} \quad (45)$$

$$r_2 \equiv \frac{\partial F(N_1(1+\Omega(\alpha,\phi(\alpha))), N_1(s^*+a_1))}{\partial K} - \delta \quad (46)$$

Again the government sets τ_{w2} and τ_{r2} so that after tax wage and after tax interest rate become the same as the ones at the initial steady state. This implies $\tau_{w2} = 1 - \frac{w^*(1-\tau_w^*)}{w_2}$, $\tau_{r2} = 1 - \frac{r^*(1-\tau_r^*)}{r_2}$. Thus, SP_2 becomes (See appendix 6)

$$SP_2 = N_1\int_{s^*}^{s^*+a_1}[\frac{\partial F(1+\Omega(\alpha,\phi(\alpha)),z)}{\partial K}+(1-\delta)]dz + N_1\int_{1+\pi_n}^{1+\Omega(\alpha,\phi(\alpha))}[\frac{\partial F(z,s^*)}{\partial L} - c^{y*} - s^* - g^y - g^{ind}]dz \quad (47)$$

Note that the first term of (47) measures the welfare gain that arises from the additional saving that the government did at the end of period 1. The second term measures the welfare gain that arises from the increased population growth rate in the presence of PYGO social security system. From (39), the inside of the first integration is positive. From the MPL condition, the inside of the second integration is positive. Thus, SP_2 is positive and the government can implement the proposed tax policy. Again, at the end of period 2, the government can use some of the above surplus to increase the supply of publicly provided private goods and use the rest to increase the balance of the government savings. Similarly, the government surplus at the period 3 becomes

$$SP_3 = N_2 \int_{s^*}^{s^*+a_2} [\frac{\partial F(1+\Omega(\alpha,\phi(\alpha)),z)}{\partial K} + 1 - \delta]dz + N_2 \int_{1+\pi_n}^{1+\Omega(\alpha,\phi(\alpha))} [\frac{\partial F(z,s^*)}{\partial L} - c^{y*} - s^* - g^y - g^{ind}]dz \tag{48}$$

where a_2 is the government saving per young population at the end of period 2. Clearly, SP_3 is positive. Again the government uses some of the surplus for increasing publicly supplied private goods and the rest for the government saving. This implies that $SP_t > 0$ for all $t = 1, 2, ...$ Thus, we have the following Proposition 2.

Proposition 2. *Consider an economy that the labor supply is elastic and that the (optimal or non-optimal) wage and interest taxes are used at the initial steady state. If MPL condition is satisfied at the initial steady state, accepting immigrants with tax rule (40) Pareto-improves welfare for all generations.*

3.4 Government Saving and the Golden Rule

This subsection examines the capital stock path and government saving path when immigrants are accepted. To examine the government saving path, we need to specify how much of the government surplus is used for the government saving . To simplify the argument, assume that the surplus that arises from the increased government saving in period $t-1$ is used for the government saving at the period t. Then, the government saving per young population at the end of period t for $t \geq 2$, a_t, becomes

$$a_t = \frac{N_{t-1}}{N_t} \int_{s^*}^{s^*+a_{t-1}} [\frac{\partial F(1+\Omega(\alpha,\phi(\alpha)),z)}{\partial K} + 1 - \delta]dz \tag{49}$$

$$= \frac{1}{1+\Omega(\alpha,\phi(\alpha))} \int_{s^*}^{s^*+a_t} \frac{\partial F(1+\Omega(\alpha,\phi(\alpha)),z)}{\partial K} + 1 - \delta]dz \tag{50}$$

From the assumption (39) that the government keeps saving until the economy reaches the golden rule level of capital stock per capita, (49) becomes

$$a_{t+1} \geq \frac{1}{1+\Omega(\alpha,\phi(\alpha))}\int_{s^*}^{s^*+a_t} \Omega(\alpha,\phi(\alpha)) + \delta + (1-\delta)]dz \tag{51}$$

$$= a_t \tag{52}$$

The equality of the above equation holds only when the economy reaches the golden rule level of capital stock per capita. Thus, a_t is increasing over time given that a_t is determined by (49).

To analyze dynamic behavior of the government saving path, it is useful to rewrite (50):

$$a_{t+1} = \frac{1}{1+\Omega(\alpha,\phi(\alpha))}[F(1+\Omega(\alpha,\phi(\alpha)), s^*+a_t) - F(1+\Omega(\alpha,\phi(\alpha)), s^*) + (1-\delta)a_t] \tag{53}$$

$$\equiv Q(a_t) \tag{54}$$

Figure 2 shows that the graph of $a_{t+1} = Q(a_t)$. $Q(a_t)$ is zero at a_t= 0 and $Q(a_t)$ is concave due to the diminishing marginal product of capital. The slope of $Q(a_t)$ at $a_t = 0$ is

$$Q'(0) = \frac{1}{1+\Omega(\alpha,\phi(\alpha))}(\frac{\partial F(1+\Omega(\alpha,\phi(\alpha)), s^*)}{\partial K} + 1 - \delta) \tag{55}$$

Because of the assumption (39), $Q'(a_t)$ at $a_t = 0$ is greater than one. Thus, the $a_{t+1} = Q(a_t)$ and 45 degree line intersect at $a_t = 0$ and a^* where $a^* > 0$. Let a^{**} be the point where $Q'(a^{**})$ = 1.This implies that

$$\frac{\partial F(1+\Omega(\alpha,\phi(\alpha)), s^*+a^{**})}{\partial K} = \delta + \Omega(\alpha,\phi(\alpha)) \ .$$

In other words, at a^{**} the golden rule is satisfied. Note that the government can choose a_1 so that $a_1 > 0$. From the graph of $a_{t+1} = Q(a_t)$, a_t keeps increasing starting from a small $a_1 > 0$. Before it reaches a^*, it reaches a^{**} within a finite time. This implies that the economy reaches the golden rule level of capital stock per capita with a finite period.

Proposition 3. (Reaching Golden rule) *Suppose that the PYGO social security system is initially used and that MPL condition is satisfied. Then, by accepting immigrants and using* *the proposed tax and government saving policy, the government can make the economy reach the golden rule level within in a finite time in a Pareto-improving way.*

Note that in the above analysis, I assume that the returns that arises from the government saving balance at the period $t-1$ is used for the government saving at the period t. However, there is an additional surplus that arises from accepted immigrants. Thus, the government can shorten the time to reach the golden rule by using the surplus arising from the accepted immigrants.

4 Quantifying the Welfare Gain of immigration in the Presence of PYGO Social Security

Proposition 1 and 2 in the previous section shows that accepting immigrants can Pareto-improve the welfare of all generations if and only if there are inter-generational transfers in the sense that the marginal product of labor of the young is greater than the sum of consumption, including publicly provided private good and saving of the young (MPL condition). Furthermore, Proposition 3 shows that if the government can save some of the welfare gain as the government saving, it is possible to make the economy reach the golden rule level of capital stock in a Pareto-improving way. This is a sharp contrast to the previous literature of the social security reform that shows that to increase the capital stock of the economy in the presence of PYGO social security system, some generation must bear the double burden(Geanakoplos, John, Olivia S. Mitchell and Stephen P. Zeldes 1998) .

There are several questions to the proposition 1, 2 and 3, however. First, those propositions are based on two period overlapping generation model. In a realistic multi-period overlapping generation model, it might not be possible for an economy to reach the golden rule level of capital stock per capita in a Pareto-improving way by accepting immigrants. Second, although the proposition 3 show that the economy reaches the golden rule level of capital stock per capita in a finite time, practically it might take a quite long time to reach the golden rule from a policy perspective although the economy reaches within in a finite time. Third, the proposition 1-3 are silent about the quantitative welfare effect. Given that it might take a quite long time to reach the golden rule, the welfare gain of accepting immigrants can be very small.

This section answers those questions. To answer those questions, I use the computational overlapping generation model developed by Auerbach and Kotlikoff and conduct the following thought experiment. First, I consider a hypothetical model economy where there is no immigrant at the initial steady state and its population growth rate is equal to the average of the predicted population growth rate of the US economy in the next 80 years. I assume that the initial steady population growth rate is 0.55 percent and that its characteristics are similar to the ones of the US.[6] I assume that the government increases the population growth rate from 0.55 percent to 0.775, 1, 1.45 or 1.9 percent by accepting immigrants in this model economy. Then, I examines whether increasing of the population growth rate in this manner Pareto-improves welfare of all cohorts in this model economy and quantify the welfare effect of this increased population growth rate.

4.1 Auerbach and Kotlikoff model with immigration

Agents live for 80 periods. From age 1 until age 45 they work. At 46, they retire. Let p_i be the probability that an agent is alive the age i given that he or she is alive until age $i-1$. Let j be the index to denote whether an agent is a native or an immigrant. $j = n$ implies that an agent is a native and $j = m$ means that an agent is an immigrant. An agent who is born at the period t maximizes the following utility function:

$$\max_{\{c^j_{t+i,i} l^j_{t+i,i}\}} \sum_{i=1}^{45} \beta^i \prod_{q=1}^{i-1} p_q \left\{ \frac{[(c^j_{t+i,i})^{\alpha}(1-l^j_{t+i,i})^{1-\alpha}]^{1-\gamma}}{1-\gamma} + g_{t+i,i} \right\} + \sum_{i=46}^{80} \beta^i \prod_{q=1}^{i-1} p_q \left\{ \frac{[c^j_{t+i,i}]^{\alpha(1-\gamma)}}{1-\gamma} + g_{t+i,i} \right\} \tag{56}$$

[6] Since the US economy has been accepting immigrants, the characteristics of the model economy is not exactly the same as the real US economy. I assume that there is no immigrants at the initial steady state so that my simulation becomes similar to the theoretical model. In another paper, I considered more explicitly a case where the characteristics of the model economy is very similar to the US in the sense that there is immigrants at the initial steady state.

where $c^j_{t+i,i}$ and $l^j_{t+i,i}$ are the amount of the private consumption and the labor supply of type j agent. $g_{t+i,i}$ is the publicly provided private good at the age i in the period $t+i$. The government does not discriminate immigrants in terms of the consumption of the publicly provided private goods. I assume that the amount of $g_{t+i,i}$ is chosen by the government.

The budget constraint of an agent at age i

$$
\begin{aligned}
s^j_{t+i-1,i-1}(1+r_{t+i}(1-\tau_{r,t+i})) + (1-\tau_{w,t+i})w_{t+i}H^j_i \times l^j_{t+i,i} &= c^j_{t+i,i} + s^j_{t+i,i} \text{ for } 1 \leq i \leq 45 \\
s^j_{t+i-1,i-1}(1+r_t(1-\tau_{rt})) + b^j_{t,i} &= c^j_{t+i,t} + s^j_{t+i,i} \text{ for } 46 \leq i \leq 80 \\
s^j_{t+i,i} &\geq 0 \qquad\qquad \text{for } 1 \leq i \leq 80
\end{aligned}
$$

where H^j_i is the efficient unit of human capital of type j(native or immigrant) and $H^j_i > 0$ for $1 \leq i \leq 45$ and $H^j_i = 0$ for $i \geq 46$. I assume that an individual cannot have a negative saving balance. Once an individual dies, the government imposes 100 percent bequest tax. $b^j_{t,i}$ is the social security benefit for type j. For $i \leq 45, b_{t,i} = 0$ and for $i \geq 46$, $b^j_{t,i}$ is determined as follows:

$$b^j_{t,i} = 12 \times RR \times AIME^j(t-i+1)$$

where RR is the replacement and AIME(t) is the average income monthly index of the cohort who is born at the year t. AIME(t) is calculated as :

$$AIME^j(t) = \frac{\sum_{i=1}^{45} w_{t+i-1}H^j_i}{45 \times 12}$$

For the production function, I assume that the economy's aggregate production can be described as Cobb-Douglas production:

$$Y_t = K^\theta_t(E_tL_t)^{1-\theta}$$

$$\mu = (E_{t+1} - E_t)/E_t$$

where θ is the capital share and E_t is a parameter to represent a technology level. μ is income per capita growth rate. L_t is the efficient unit of labor supply at the period t. L_t is defined as follows:

$$L_t = \sum_{i=1}^{45} \sum_{j=n,m} H_i^j N_{t,i}^j$$

where $N_{t,i}^j$ is the population of age i of type j at the period t. Given the probability that an agent of age $i-1$ is alive at $i-1$, for $i \geq 2$:

$$N_{t,i}^j = N_{t-1,i-1}^j p_{i-1} \qquad \text{where } i \geq 2$$

The government budget constraint at the initial steady state is

$$\begin{aligned} \tau_{wt} w_t L_t + \tau_{rt} r_t \sum_{i=1}^{80} p_i N_{t-1,i}^n s_{t-1,i-1}^n + (1+r_t) \sum_{i=1}^{80} (1-p_i) N_{t-1,i}^n s_{t-1,i-1}^n \\ - \sum_{i=1}^{80} N_{t,i}^n g_i^* (1+\mu)^t - \sum_{i=46}^{80} b_{t,i}^n \times N_{t,i}^n = 0 \end{aligned} \tag{57}$$

As for immigrants, I assume that all immigrants will come to the host country at age 0. I assume that at the initial steady state, the population growth rate of the native be π_n. This implies that

$$N_{t+1,1}^n = (1+\pi_n) N_{t,1}^n$$

To consider the population growth rate with immigration, it is useful to deified cohort-to-cohort population growth rate and year-to-year population growth rate. Let $\overline{N}_{t,1} = N_{t,1}^n + N_{t,1}^m$. The cohort-to-cohort population growth rate

$$\text{cohort-to-cohort population growth rate } = \frac{\overline{N}_{t+1,1} - \overline{N}_{t,1}}{\overline{N}_{t,1}}. \tag{58}$$

The period to period population rate is

$$\text{period-to-period population growth rate } = \frac{\sum_{i=1}^{80} \overline{N}_{t+1,i} - \sum_{i=1}^{80} \overline{N}_{t,i}}{\sum_{i=1}^{80} \overline{N}_{t,i}} \tag{59}$$

For immigration policy, let π_{target} be the target population growth rate. In one scenario, I assume that the government accepts immigrants from year 1 and choose $N_{t,1}^m$ as follows:

$$\pi_{\text{target}} = \frac{\overline{N}_{t+1,1} - \overline{N}_{t,1}}{\overline{N}_{t,1}} \tag{60}$$
$$\text{where } \overline{N}_{t,1} = N_{t,1}^n + N_{t,1}^m$$

This scenario implies that from period 1,the cohort-to-cohort population growth rate reaches the target population growth rate π_{target}. But it takes 81 years that the year-to-year population growth rate reaches the target population growth rate. To shorten years in which the year-to-year population growth rate reaches the target population growth rate, another scenario can be considered. Let T^* be the calendar year in which the year-to-year population growth rate reaches the target population growth rate. Then, the immigrants are accepted to according to

$$\frac{t}{T^*} \times \pi_{\text{target}} = \frac{\sum_{i=1}^{80} \overline{N}_{t+1,i} - \sum_{i=1}^{80} \overline{N}_{t,i}}{\sum_{i=1}^{80} \overline{N}_{t,i}} \text{ for } 1 \leq t \leq T^* \tag{61}$$
$$\pi_{\text{target}} = \frac{\sum_{i=1}^{80} \overline{N}_{t+1,i} - \sum_{i=1}^{80} \overline{N}_{t,i}}{\sum_{i=1}^{80} \overline{N}_{t,i}} \text{for } T^* + 1 \leq t \tag{62}$$
$$\text{where } \quad \overline{N}_{t,1} = N_{t,1}^n + N_{t,1}^m \text{ and } N_{t,i}^j = N_{t-1,i-1}^j p_{i-1} \quad \text{for } i \geq 2$$

Note that the RHS of (61) is the year-to-year population growth rate. Thus, (61) implies that the at year t, the year-to-year population growth rate is t/T^*. At period T^*, the year-to-year population growth rate reaches the target population growth rate π_{target} and after year T^*, immigrant is accepted so that the year-to-year population growth rate is equal to the target population growth rate.

The capital stock at the period t is the sum of the balance of individual savings and government saving. Let a_{t-1} be the balance of the government saving per capita at the end of period $t-1$.

Then, the total capital stock at the period t is

$$K_t = \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1,i}^{j} s_{t-1,i}^{j} + a_{t-1} \sum_{i=1}^{80} \overline{N}_{t-1,i}$$

The efficient unit wage rate at time t, w_t and the pre-tax interest rate at time t, r_t, are determined as

$$w_t = (1-\theta) K_t^{\theta} E_t^{1-\theta} L_t^{-\theta} \qquad \text{and} \qquad r_t = \theta K_t^{\theta-1} E_t^{1-\theta} L_t^{1-\theta}$$

When the government accepts immigrants, the wage rate decreases and the interest rate increases. To Pareto-improve welfare, I assume that the government keeps the after tax wage rate and interest rate the same as at the initial balance growth path. More specifically, let τ_{wt} and τ_{kt} be the wage tax rate and the interest tax rate after accepting immigration, respectively. Let w_t^*, r^*, τ_w^*, τ_k^* be the efficient unit wage rate and the interest rate, the wage tax rate and the interest rate at the initial balanced growth path, respectively. Let w_t and r_t be the efficient unit wage rate and the interest rate at the time t. Then, the wage tax rate τ_w and interest tax rate τ_k are set as follows:

$$w_t(1-\tau_{wt}) = w_t^*(1-\tau_w^*) \text{ and } r_t(1-\tau_{rt}) = r^*(1-\tau_r^*) \tag{63}$$

When the taxes are set according to equation (63) , there is a surplus for the government budget even if the government spends the same amount of publicly provided private good per person as at the initial balance growth path. The government can use this surplus for increasing the government saving or increasing the level of publicly provided good. Let Ω be the distributional parameter that indicates how much percentage of the budget surplus is used for the government saving. Then, the balance of the government saving at the period t is

$$a_t \sum_{i=1}^{80} \overline{N}_{t,i} = a_{t-1} \sum_{i=1}^{80} \overline{N}_{t-1,i} + \Omega \times SP_t \tag{64}$$

$$\begin{aligned} SP_t = \tau_{wt} w_t L_t + \tau_{rt} r_t \sum_{i=1}^{80} \sum_{j=n,m} p_i N^j_{t-1,i} s^j_{t-1,i-1} + (1+r_t) \sum_{i=1}^{80} \sum_{j=n,m} (1-p_i) N^j_{t-1,i} s^j_{t-1,i-1} \\ - \sum_{i=1}^{80} \overline{N}_{t,i} g^*_i (1+\mu)^t - \sum_{i=46}^{80} \sum_{j=n,m} B^j_{t,i} \times N^j_{t,i} \end{aligned} \tag{65}$$

The rest of the government surplus, $(1-\Omega) \times SP_t$ is used to increase the amount of publicly provided private good:

$$\sum_{i=0}^{80} \overline{N}_{t,i} \widetilde{g}_{t,i} = (1-\Omega) \times SP_t \tag{66}$$

I assume that the ratio of the increased amount of the publicly provided private good are same for all ages:

$$\frac{\widetilde{g}_{t,i}}{g^*_i (1+\mu)^t} = \text{constant for all } i \tag{67}$$

where $g^*_i (1+\mu)^t$ is the amount of publicly provided private good at the initial balanced growth path. Then, the amount of publicly provided private goods after the acceptance of immigrants becomes

$$g_{t,i} = g^*_i (1+\mu)^t + \widetilde{g}_{t,i}. \tag{68}$$

The efficient unit wage rate and the interest rate are determined as follows:

$$w_t = E_t (1-\theta) K^\theta_t (L_t E_t)^{-\theta}$$

$$r_t = \theta K^{\theta-1}_t (E_t L_t)^{1-\theta} - \delta$$

4.2 Thought Experiment and Parameters Values for Simulation

For applying the computational overlapping generation model, I consider the following thought experiment. Currently the US census bureau projects the future population, including international immigrants, in the next 100 years assuming that the current immigration policy is maintained. It projects that the averaged future population growth rate in the next 100 years is 0.55 percent. First, I assume that the US economy is on the balanced growth path with this projected population growth rate, 0.55. Also, in order to be consistent with the theoretical part, I assume that there is no immigrant at the initial steady state. Then, I will consider hypothetical situations where this population growth rate is increased to 0.775, 1, 1.45, 1.9 percent due to immigration. Given those hypothetical situations, I calculate how long it takes for the economy to reach the golden rule level in a Pareto-improving way and evaluate the increased utility.

More specifically, each parameters are obtained as follows. As for the population growth rate, the US census bureau published a population projection (with international migration) from 2000 to 2100. I calculate the annual population growth rate(with international immigration) at the age of 30 from 2020 to 2100 by using the table published by the Census Bureau. Then, I averaged those annual population growth rates and obtained 0.55 percent as the averaged annual population growth rates between 2020 and 2100. Thus, I use 0.55 percent as an initial population growth rate in the model economy. To calculate a reasonable increased population growth rate due to immigration I look at the population projection without international immigration, which is published from the census bureau and calculate the average population growth rates without international immigration from 2020 to 2100. I averaged those annual population growth rates and it becomes 0.1 percent. This implies that with the current immigration policy, the population growth rate increases by 0.45 percentage point. Thus, I consider four cases where the population growth rate is increased from 0.55 percent to 0.75 percent ($0.55+0.5\times0.45$), 1 percent ($0.55+0.45$), 1.45 percent($0.55 + 2 \times 0.45$), 1.9 percent ($0.55 + 3 \times 0.45$) , respectively.

For the age specific government expenditure, g_i, I follow Storesletten (1995) and Auerbach, Kotlikoff Hagememann and Nicoletti (1989). I assume that g_y, g_m and g_o are =24.5% ,13.4% and 23.2 % of GDP per capita at the initial steady state.

For the coefficient the relative risk aversion, γ, the literature has not found the precise estimate. Auerbach and Kotlikoff and Storesletten assumed that it is 4. Nishiyama and Smetter (2007) set $\gamma = 2$. Following Storesletten, I assume that $\gamma = 3$ and check the robustness with different value of γ. For β, following Hurd, Rios-Rull and Storesletten, I assume that $\beta = 1.011$. This will generate a reasonable level of the marginal product of capital, around 10 percent, at the initial steady state. For the leisure share in the utility function, α, it is assumed that it is 0.33.

As for the production side, the depreciation rate, δ , I assume that $\delta = 0.047$ according to Storesletten. For the capital share in the production function, θ, I set $\theta = 0.33$. For technological progress, I assume that μ, income per capita growth rate, is 1.5 following Storesletten.

For the human capital profile of the native, H_i^s, I take the value from Auerbach and Kotlikoff.

$$H_i^n = \exp(4.47 + 0.033 \times i - 0.00067 \times i^2) \quad \text{for } 1 \leq i \leq 45 \tag{69}$$

$$H_i^n = 0 \quad \text{for } 46 \leq i \tag{70}$$

For the human capital profile of the immigrants, I calculate from Storesletten[7] and assume that $H_i^m = 0.834 \times H_i^n$. For the value of p_i, I take the values from Nishiyama and Smetter (2007).

For the capital income tax, I take the value from Nishiyama and Smetter (2007) and it is assumed that $\tau_k = 0.28$. For the social security benefit, following Auerbach and Kotlikoff, I determine the benefit level such that the replacement ratio becomes 0.6 and check the robustness of the results by varying it from 0.6 to 0.55 and 0.5.

[7] Figure 2.2 of Storesletten shows that at age 20,25, 30,35, 40,45, the wage rate of the immigrant is lower than native by 15%, 20%, 17.8%, 19.1%, 14.4% and 13% respectively. By averaging those rates, I obtained 16.6%.

4.3

4.4 Results

Figure 3, 4 and 5 show the life-cycle asset balance, consumption and leisure of an consumer at the initial balanced growth path. In those graphs, the period-to-period population growth rate is set to 0.55 percent and CRRA is set to 3. At the age of 46, the consumption of leisure becomes 1 due to the mandatory retirement. Figure 6 shows year to year population growth rate when immigrants are accepted according to (60) and the target population growth rate is set 0.775 percent. This implies that from year 1 the cohort to cohort population growth rate becomes 0.775 percent. However, it takes 81 years for the year to year population growth rate reaches 0.775 percent. Figure 7 shows the path of capital output ratio when the population growth rate is increased from 0.55 to 0.775 due to immigration. The initial capital output ratio in my simulation is 3.35. In the business cycle literature, 2.94 is used frequently (Coolely and Prescott(1995)). My simulation result shows that it is not far awary from this reference value. Figure 8 show the maginal product of capital, which is negatively related with capital labor ratio. The capital labor ratio decreases initially since the economy has more labor due to immigrants. However, as the government saving increases, the capital labor ratio starts to increase around 30 years and keeps increasing until the economy reaches the golden rule level. This pattern of the change of the capital stock over time is observed in all cases whenever the economy is Pareto-improved. In Figure 8, it is observed that around 180 years the maginal product of capital reaches the golden rule level. Figure 9 compares the utility level at the initial balanced growth path with the utility level with the acceptance of immigrants when the distributional share, Ω in equation (64), is 100 percent . It implies that all the surplus is used for the government saving until the economy reaches the golden rule level and the surplus is distributed to consumers only after the economy reaches the golden rule level. This implies that the utility of the cohort 190 starts to be higher than at the initial balanced growth path. Figure 10 shows the increased utility by the equivalent variation.

Table 1, 2 and 3 are the parameter values and the results of the simulation. In Table 1, immigrants are accepted according to (60). As can be seen, the year-to-year population growth rate reaches the target population growth rate in 81 years. The distributional share in table 1 is Ω in equation (64). It shows how much percentage of the surplus obtained by accepting immigrants is used for the government saving for future cohorts. Columns 8-12 are the result of the simulation. Column (8) shows how many years it takes for the economy to reach the golden rule level in a Pareto-improving way by increasing the population growth rate. For example, in the benchmark case, case 1, it takes 187 years for the economy to reach the golden rule level of the capital stock per capita in a Pareto-improving way by increasing the population growth rate from 0.55 percent to 0.775 percent through accepting immigrants. Column (9) shows how much capital stock per capita increases at a new balanced growth path compared with the initial balanced growth path. In the case 1, for example, it shows that the capital stock per capita increases by 77 percent through accepting immigrants. Column (10) shows how much percentage the publicly provided private good per capita increases at this new balanced growth path. In the analysis, it is assumed that the government adjusts the wage tax rate and interest tax rate so that after tax wage rate and interest rate are the same as at the initial balanced growth path. This implies that Pareto-improvement is achieved though the increase of publicly provided private good. Column (10) shows this quantity of the increased publicly provided private good per capita at the new balanced growth path. Column (11) shows how much percentage the utility, measured by the expenditure function, of the cohort who are born at the new balanced growth path increases compared with the utility at the initial balanced growth path. In the bench mark case, case 1, the utility level measured by the expenditure function increases by 2.01 percent at the new balanced growth path. Column (12) shows the present discounted value of the increased utility of all cohorts, measured by the expenditure function. In the expenditure function, the price vector at the initial steady state is used for evaluating the utility. In the bench mark case, the present discounted value of the utility of the all cohorts of the native will increase by 0.014 percent.

Cases 2-16 in Table 1 show the simulation with other parameter values. From case 2 to case 4, the target population growth rate is set, 1, 1.45, 1.9 percent, respectively. The simulation shows that as the government accepts more immigrants, the years taken to reach the golden rule level becomes shorter and the present discounted value of the increased utility become higher. In case 5-8, the distributional share, Ω, is decreased from 1.0 to 0.9. This means that 90 percent of the surplus is used for the government saving and 10 percent of the surplus is used to increase the publicly provided private good of the current cohort. Thus, the utility of the cohort starts to be strictly higher than the level at the initial balanced growth path even before the economy reaches the golden rule level. In those cases, the government uses less for the government saving since 10 percent of the surplus is distributed to the current cohort. This implies that it takes more years to reach the golden rule level of capital stock per capita. Although it takes more years to reach the golden rule level, the present discounted increased utility of all cohort is higher than in the case 1-4. This is due to the fact that the utility of the all cohort starts to increase even before the economy reach the golden rule level[8].

Cases 9-12 and 13-16 analyzed scenarios where the distributional share is 80 percent and 70 percent, respectively. As the distributional share for the government saving becomes lower, the longer it takes for the economy to reach the golden rule level while the present discounted utility increases.

Table 2 and Table 3 conducted robustness checks. To do so, first in Table 2, I increased the speed of accepting immigrants. In those cases, immigrants are accepted according to (60) and T^* is set to 40. Figure 10 shows the year to year population growth rate in this case. As the Figure

[8]In case 1-4, the utility of the cohort starts to increase after the economy reaches the golden rule level.

10 shows, the year-to-year population growth rate reaches the target population growth rate in 40 years. The table 2 shows that the quantitative result are almost the same as the result in the Table 1. The years to taken to reach the golden rule level of capital stock are almost the same as in the Table 1 and the present discounted value of the increased utility values.

In Table 3, I changed the parameter values of CRRA and the replacement rate. First, the value of CRRA is changed from 3 to 2 or 4. When CRRA is equal to 2, the present discounted value of increased utility becomes higher and years taken to reach the golden rule level of capital stock becomes shorter in most cases. When CRRA is equal to 4, the years taken to reach the golden rule level becomes longer and the present discounted value of the utility becomes smaller. In addition, when the target population growth rate is equal to 1.9, it was impossible to Pareto-improve welfare of all cohort by increasing the size of immigrants. In case 8 of Table 3, the capital stock per capita kept decreasing and reached zero at one point. Case 9-12 and 13-16 decreased the replacement from 0.6 to 0.55 and 0.5, respectively, As the replacement rate becomes lower, the less the degree of the intergenerational government transfer. Theoretically, it implies that the positive welfare effect of accepting immigrant becomes smaller as the Proposition 1 shows. This intuition is verified on case 9-16. As the replacement rate becomes smaller, the years taken to reach the golden rule becomes longer and the present discounted value of the increased utility becomes smaller.

Summary of the Simulation Results

The simulation considered cases where the population growth rate is increased from 0.55 percent to 0.775 percent, 1 percent, 1.45 percent, 1.9 percent due to immigration (from zero immigration to positive immigration).

1) By increasing the population growth rate from 0.55 percent to 0.775 percent by accepting immigrants, it is possible to Pareto-improve welfare all cohorts and making the economy reach the golden rule level of capital stock per capita. The result is almost similar when the target

population growth rate is change from 0.775 percent to 1, 1.45 percent and 1.9 percent except the results regarding the years taken to reach the golden rule level of capital stock.

2) In most cases, it takes 150-200 years to reach the golden rule level in a Pareto-improving way by accepting immigrants. In the new balanced growth path, the capital stock per capita increases by around 30 percent to 70 percent and the publicly provided private good per capita increases by around 19 percent.

3) The present discounted increased utility of all cohort will be around 0.014 percent to 0.05 percent.

4) For some parameter values of the utility function and the replacement, it is not possible to Pareto-improve welfare when the size of the increased immigrants is large. But even for those parameter values, a modest increase of immigrants Pareto-improve welfare and makes the economy reach the golden rule level of capital stock per capita.

5 Conclusion

In this paper, I have examined the welfare effect of accepting immigrants in the presence of a PYGO social security system qualitatively and quantitatively. First I have shown that if and only if there are inter-generational transfers from the young to the old in the sense that the marginal product of labor of the young is higher than what the young receive while they are young, accepting immigrants is Pareto-improving. Second, I have shown that if the government adjusts the wage tax rate and the interest tax rate so that the after-tax wage rate and the interest rate tax rate after accepting immigrants are same as at the initially steady state, Pareto-improvement is achieved. Third, I have shown that by accepting immigrants, it is possible to make the economy reach the golden rule level of capital stock per capita within a finite time in a Pareto-improving way. Fourth, using the Auerbach-Kotlikoff computation dynamic overlapping generation model, I calculated years taken to reach the golden rule level of capital stock per capita and the degree of

Pareto-improvement when the government increases immigrants with several parameter values. In most cases, the analysis shows that it take 150 to 200 years for the economy to reach the golden rule level of capital stock by accepting a reasonable size of immigrants. At the new steady state, welfare level is 2.2 percent higher that at the initial steady state, measured by the expenditure function. However, since it took 150-200 years to reach golden level of capital stock per capita, the present discounted value of the increased utility of all cohort is not so much. In most cases, the present discounted value of the increased utility of all cohort, measured by the expenditure function, 0.02 percent.

Appendices

Appendix 1

Proof.

The government budget constraint at the period t is

$$\tau_{w1} w_t N_t + \tau_{rt} s_{t-1} N_{t-1} = N_{t-1}(b + g^o) + N_t g^y + g^{ind}\{N_t + N_{t-1}\} \tag{71}$$

From the individual budget constraint,

$$w_t(1 - \tau_{wt}) = c_t^y + s_t$$

Solving t_{wt}, we have

$$\tau_{wt} = 1 - \frac{c_t^y + s_t}{w_t}$$

Substituting t_{wt} into the government budget constraint, we have

$$\{w_t - c_t^y - s_{t-1}\}N_t + \tau_{rt} r_t s_{t-1} N_{t-1} + (1 + r_t) a_{t-1} N_{t-1} = N_{t-1}(b + g^o) + N_t g^y + g^{ind}\{N_t + N_{t-1}\} + a_t N_t$$

$$w_t N_t + \tau_{rt} r_t s_{t-1} N_{t-1} + (1 + r_t) a_{t-1} N_{t-1} = \{c_t^y + s^t + a_t + g^y\}N_t + N_{t-1}(b + g^o) + g^{ind}\{N_t + N_{t-1}\}$$

By using homogeneity of production function and Euler's theorem, we have

$$F(N_t, (s_{t-1} + a_{t-1})N_{t-1}) - \frac{\partial F}{\partial K}(s_{t-1} + a_{t-1})N_{t-1} + \tau_{rt} r_t s_{t-1} N_{t-1} + (1 + r_t)a_{t-1}N_{t-1}$$
$$= \{c_t^y + s_t + g^y\}N_t + N_{t-1}(b + g^o) + g^{ind}\{N_t + N_{t-1}\}$$
$$F(N_t, (s_{t-1} + a_{t-1})N_{t-1}) - (r_t + \delta)(s_{t-1} + a_{t-1})N_{t-1} + \tau_{rt} r_t s_{t-1} N_{t-1} + (1 + r_t)a_{t-1}N_{t-1}$$
$$= \{c_t^y + s^t + a_t + g^y\}N_t + N_{t-1}(b + g^o) + g^{ind}\{N_t + N_{t-1}\}$$
$$F(N_t, (s_{t-1} + a_{t-1})N_{t-1}) + (1 - \delta)(s_{t-1} + a_{t-1})N_{t-1}$$
$$= \{c_t^y + s^t + a_t + g^y\}N_t + N_{t-1}[s_{t-1}(1 + r_t(1 - t_{rt}) + b + g^o)] + g^{ind}\{N_t + N_{t-1}\}$$

Since $s_{t-1}(1 + r_t(1 - t_{rt})) + b = c_t^o$, we have

$$F(N_t, (s_{t-1}+a_{t-1})N_{t-1})+(1-\delta)(s_{t-1}+a_{t-1})N_{t-1} = \{c_t^y+s^t+a_t+g^y\}N_t+N_{t-1}\{c_t^o+g^o\}+q\{N_t+N_{t-1}\}$$

Appendix 2

Consider the following programming problem

$$\max_{\{c_t^o, c_t^y, s_t | t=1,2...\}} \frac{1}{1+\rho}[u_o(c_1^o) + v_o(g^o, q)] + \sum_{t=1}^{\infty} \beta^t U(c_t^y, c_{t+1}^o, g^y, g^o, q, q)$$
$$\text{s.t.} \quad F(N_t, N_{t-1}s_{t-1}) + (1 - \delta)N_{t-1}s_{t-1} - N_t(c^y + s_t + g^y)$$
$$-N_{t-1}(c^o + g^o) - q\{N_t + N_{t-1}\} = 0 \text{ for } t = 1, 2, ...$$
$$\text{where } s_0 = s^*$$

where g^y, g^o, q are constant and given. Note that if the steady state consumption and saving are the solution of the above programming problem with the constraint that there is no immigration, then it implies that the steady state equilibrium allocation is Pareto-efficient with the constraint that there is no immigration. Now define the Lagrangian function as follows:

$$L = \frac{1}{1+\rho}[u_o(c_1^o) + v_o(g^y, q)] + \sum_{t=1}^{\infty} \beta^t U(c_t^y, c_{t+1}^o, g^y, g^o, q, q)$$
$$+ \sum_{t=1}^{\infty} \lambda_t \{F(N_t, N_{t-1}s_{t-1}) + (1-\delta)N_{t-1}s_{t-1} - N_t(c^y + s_t + g^y) - N_{t-1}(c^o + g^o) - q(N_t + N_{t-1})\}$$

The firsts order conditions of the programming problem are

$$c_t^o : \beta^{t-1} \frac{1}{1+\rho} u_o'(c_t^o) - \lambda_t N_{t-1} = 0$$
$$c_t^y : \beta^t u'(c_t^y) - \lambda_t N_t = 0$$
$$s_t \qquad -\lambda_t N_t + \lambda_{t+1} N_t (1-\delta) + \lambda_{t+1} N_t \frac{\partial F}{\partial K} = 0$$

Transversality condition implies that

$$\lim \beta^t u_y'(c_t^y) s_t \to 0$$

From those first order condition, we have

$$1 - \delta + \frac{\partial F}{\partial K} = \frac{\lambda_t}{\lambda_{t+1}}$$
$$\beta(1+\rho) \frac{u_y'(c_t^y)}{u_o'(c_t^o)} = 1 + \pi_n$$
$$\beta = \frac{\lambda_t}{\lambda_{t-1}}(1 + \pi_n)$$

Now suppose that we set

$$\frac{\lambda_t}{\lambda_{t+1}} = 1 + r^*$$
$$\beta = \frac{1+\pi_n}{1+r^*}$$
$$c_t^y = c^{y*} \text{ and } c_t^o = c^{o*}$$
$$s_t = s^*$$

Then, the first order condition of the programming problem and the transversality condition is satisfied as long as $r^* > \pi_n$ which is guaranteed from the assumption of dynamic efficiency. Also, the from the definition of the steady state condition, the resource constraint is satisfied. Therefore, the steady state allocation is Pareto-efficient allocation. QED

Appendix 3

Proof of observation 2

Let the solution of the programming problem for α_1 be $\{c_t^y(\alpha_1), s_t(\alpha_1), c_t^o(\alpha_1); t = 1, 2, 3, ...\}$ and let the solution for α_2 be $\{c_t^y(\alpha_2), s_t(\alpha_2), c_t^o(\alpha_2); t = 1, 2, 3, ...\}$. By definition, the solution must satisfy the resource constraint:

$$F(1+\Omega(\alpha_1), s_{t-1}(\alpha_1)) - (1-\delta)s_{t-1}(\alpha_1) - \{c_t^o(\alpha_1) + g^o + g^{ind}\} - (1+\Omega(\alpha_1))\{c_t^y(\alpha_1) + g^y + g^{ind}\} = 0$$

$$F(1+\Omega(\alpha_2), s_{t-1}(\alpha_2)) - (1-\delta)s_{t-1}(\alpha_2) - \{c_t^o(\alpha_2) + g^o + g^{ind}\} - (1+\Omega(\alpha_2))\{c_t^y(\alpha_2) + g^y + g^{ind}\} = 0$$

Then, consider the solution of the programming problem for $\alpha_3 \equiv \theta\alpha_1 + (1-\theta)\alpha_2$. Let the solution for α_3 be $\{c_t^y(\alpha_3), s_t(\alpha_3), c_t^o(\alpha_3); t = 1, 2, 3, ...\}$. Then, Now consider, another consumption and saving path: $\{\theta c_t^y(\alpha_1)+(1-\theta)c_t^y(\alpha_2), \theta s_t(\alpha_1)+(1-\theta)s_t(\alpha_2), \theta c_t^o(\alpha_1)+(1-\theta)c_t^o(\alpha_2); t = 1, 2, 3, ...\}$. Note that this consumption and saving path satisfies the minimum utility constraint from the concavity of the utility function. Also, check whether this consumption and saving path satisfies the resource constraint at period t for α_3 as shown below:

$$\begin{aligned}
&F(1+\Omega(\alpha_3), \theta s_{t-1}(\alpha_1) + (1-\theta)s_{t-1}(\alpha_2)) - (1-\delta)\{\theta s_{t-1}(\alpha_1) + (1-\theta)s_{t-1}(\alpha_2)\} \\
&- \{\theta c_t^o(\alpha_1) + (1-\theta)c_t^o(\alpha_2) + g^o + g^{ind}\} - (1+\Omega(\alpha_3))\{\theta c_t^y(\alpha_1) + (1-\theta)c_t^y(\alpha_2) + g^y + g^{ind}\} \\
&> \theta[F((1+\Omega(\alpha_1)), s_{t-1}(\alpha_1)) - (1-\delta)s_{t-1}(\alpha_1) - \{c_t^o(\alpha_1) + g^o + g^{ind}\} - (1+\Omega(\alpha_3))\{c_t^y(\alpha_1) + g^y + g^{ind}\}] \\
&+ (1-\theta)[F((1+\Omega(\alpha_2)), s_{t-1}(\alpha_2)) - (1-\delta)s_{t-1}(\alpha_1) - \{c_t^o(\alpha_2) + g^o + g^{ind}\} - (1+\Omega(\alpha_3))\{c_t^y(\alpha_2) + g^y + g^{ind}\} \\
&= 0
\end{aligned}$$

Thus, the consumption and saving path $\{\theta c_t^y(\alpha_1)+(1-\theta)c_t^y(\alpha_2), \theta s_t(\alpha_1)+(1-\theta)s_t(\alpha_2), \theta c_t^o(\alpha_1)+(1-\theta)c_t^o(\alpha_2); t=1,2,3,...\}$ satisfies the resource constraint

Because $\{c_t^y(\alpha_3), s_t(\alpha_3), c_t^o(\alpha_3); t=1,2,3,...\}$ is the solution for α_3, we have

$$
\begin{aligned}
V(\alpha_3) &\geq \frac{1}{1+\rho}[u_o(\theta c_1^o(\alpha_1)+(1-\theta)c_1^o(\alpha_2))+v_o(g^o, g^{ind})] \\
&> \theta\{\frac{1}{1+\rho}[\theta u_o(c_1^o(\alpha_1))+v_o(c_1^o(\alpha_1))\}+(1-\theta)\{\frac{1}{1+\rho}[\theta u_o(c_1^o(\alpha_2))+v_o(g^o, g^{ind})]\} \\
&= \theta V(\alpha_1)+(1-\theta)V(\alpha_2)
\end{aligned}
$$

Thus, $V(\alpha)$ is concave. Q.E.D.

Appendix 4

Notice that in the programming problem, objective function is concave and the constrained set is convex. Thus, if some allocation satisfies the first order condition, it is also the solution of the programming problem. Now set up the Lagrangian function as follows:

$$
\begin{aligned}
L &= \frac{1}{1+\rho}[u_o(c_1^o)+v_o(g^o, q)] \\
&+\sum_{t=1}^{\infty}\gamma_t\{U(c_t^y, c_{t+1}^o, g^y, g^o, g^{ind}, g^{ind})-u^*\} \\
&+\sum_{t=1}^{\infty}\lambda_t\{F(1+\pi_n, s_{t-1}+a_{t-1})+(1-\delta)s_{t-1} \\
&\quad -(c_t^o+g^o+g^{ind})-(1+\pi_n)\times(c_t^y+s_t+a_t+g^y+g^{ind})\ \}
\end{aligned}
$$

The first order conditions are:

$$c^o_1 : \frac{1}{1+\rho} u'_o(c^o_1) = \lambda_1$$

$$c^o_{t+1} : \gamma_t \frac{1}{1+\rho} u'_o(c^o_t) = \lambda_{t+1}$$

$$c^y_t : \gamma_t u'_y(c^y_1) = \lambda_t(1+n)$$

$$a_t : \lambda_{t+1}\{\frac{\partial F}{\partial K} + 1 - \delta\} = \lambda_t(1+n)$$

$$\gamma_t : U(c^y_t, c^o_{t+1}, g^y, g^o, q) - u^* = 0$$

$$\lambda_t : F(1+\pi_n, s_{t-1}+a_{t-1}) + (1-\delta)(s_{t-1}+a_{t-1}) - (c^o_t + g^o + q) - (1+\pi_n) \times (c^y_t + s_t + a_t + g^o + q)\ \} = 0$$

Now set the c^y_t, c^o_{t-1}, s_t, a_t, λ_t, γ_t as follows:

$$c^o_t = c^{o*}$$

$$c^y_t = c^{y*}$$

$$s_t = s^*$$

$$a_t = 0$$

$$\lambda_1 = \frac{1}{1+\rho} u'_c(c^{o*})$$

$$\lambda_{t+1} = \lambda_t \frac{1+\pi_n}{1+r^*}$$

$$\gamma_t \frac{1}{1+\rho} u'(c^{o*}) = \lambda_{t+1}$$

When we set c^y_t, c^o_{t+1}, s_t, a_t, λ_t, γ_t in this way, it clearly satisfies the first order condition. Since the relaxed problem is the convex programming problem, the it is the solution of the relaxed programming problem. Now notice that when c^y_t, c^o_{t+1}, s_t are set in this way, it also satisfies the intertemporal condition by the definition of stationary equilibrium. Therefore, it is also the solution of the main problem.

Appendix 5

The net government budget surplus at the period 1 is

$$
\begin{aligned}
& SP_1 = w_1 N_1 - w^* N_1 (1 - \tau_w^*) \\
& + r_1 s^* N_0 - r^* s^* N_0 (1 - t_r^*) - N_0 (b + g^o + g^{ind}) - N_1 (g^y + q) \\
& = w_1 N_1 - w^* N_0 (1 + \pi_n + \Omega(\alpha, 0) - \pi_n)(1 - \tau_w^*) \\
& + r_1 s^* N_0 - r^* s^* N_0 (1 - \tau_r^*) - N_0 (b + g^o + g^{ind}) - N_0 (1 + \pi_n + \Omega(\alpha, 0) - \pi_n)(g^y + g^{ind}) \\
& = w_1 N_1 - w^* N_0 (1 + \pi_n)(1 - \tau_w^*) + w^* N_0 (\Omega(\alpha, 0) - \pi_n)(1 - \tau_w^*) \\
& + r_1 s^* N_0 - r^* s^* N_0 (1 - \tau_r^*) - N_0 (b + g^o + g^{ind}) - N_0 (1 + \pi_n + \Omega(\alpha, 0) - \pi_n)(g^y + g^{ind}) \quad (72)
\end{aligned}
$$

Note that from the budget constraint at the initial steady state, we have

$$
\tau_w^* w^* N_0 (1 + \pi_n) + r^* s^* N_0 \tau_r^* = N_0 (b^* + g^o + q) + N_0 (1 + \pi_n) \times (g^y + q) \quad (73)
$$

Also, note that from the homogeneity of production function and Euler's theorem, we have

$$
\begin{aligned}
& w_1 N_0 (1 + \Omega(\alpha, 0)) + r_1 s^* N_0 = F(N_0 (1 + \Omega(\alpha, 0)),\ s^* N_0) - \delta s^* N_0 \\
& \text{and } w^* N_0 (1 + \pi_n) + r^* s^* N_0 = F(N_0 (1 + \pi_n), s^* N_0) - \delta s^* N_0
\end{aligned}
$$

Thus, the government surplus at period 1 becomes

$$
SP_1 = N_0 \int_{1+\pi_n}^{1+\Omega(\alpha,0)} [\frac{\partial F(z, , s^* N_0)}{\partial L} - w^* (1 - t_w) - g^y - q] dz
$$

$$F(N_0(1+\Omega(\alpha,0),\ s^*N_0) - F(N_0(1+\pi_n), s^*N_0) - (\Omega(\alpha,0)-\pi_n)w^*(1-t_w)N_0 - (\Omega(\alpha,0)-\pi_n)N_0(g^y+q) \tag{74}$$

$$= N_0F(1+\Omega(\alpha,0),\ s^*) - F(1+\pi_n, s^*) - (\Omega(\alpha,0)-\pi_n)w^*(1-t_w) - (\Omega(\alpha,0)-\pi_n)(g^y+q) \tag{75}$$

$$= N_0\int_{1+\pi_n}^{1+\Omega(\alpha,0)}[\frac{\partial F(z,,s^*N_0)}{\partial L} - w^*(1-t_w) - g^y - g^{ind}]dz \tag{76}$$

By the definition of the consumer's budget constraint at the initial steady state $w^*(1-\tau_w^*) = c^{y*}+s^*$, the net government surplus per worker of young at period 1 becomes

$$SP_1 = N_0\int_{1+\pi_n}^{1+\Omega(\alpha,0)}[\frac{\partial F(z,,s^*)}{\partial L} - c^{y*} - s^* - g^y - g^{ind}]dz \tag{77}$$

Since $\partial F/\partial L$ is homogenous degree zero, the above equation becomes

$$= N_0\int_{1+\pi_n}^{1+\Omega(\alpha,0)}[\frac{\partial F(zN_0,,N_0s^*)}{\partial L} - c^{y*} - s^* - g^y - g^{ind}]dz \tag{78}$$

Appendix 6

$$SP_2 = w_2\tau_{w2}N_2 + r_2\tau_{r2}s^*N_1 - N_1 \times (b^* + g^o + g^{ind}) - N_2 \times (g^y + g^{ind}) + (1+r_2)a_1N_1$$

$$= w_2N_1 - w^*N_2(1-\tau_w^*)$$

$$+ r_2s^*N_1 - r^*s^*N_1(1-\tau_r^*) - N_1(b + g^o + g^{ind}) - N_2(g^y + g^{ind}) + (1+r_2)a_1N_1$$

$$= w_2N_2 - w^*N_1(1+\pi_n + \Omega(\alpha,\phi(\alpha)) - \pi_n)(1-\tau_w^*)$$

$$+ r_2s^*N_1 - r^*s^*N_1(1-\tau_r) - N_1(b + g^o + g^{ind})$$

$$- N_1(1+\pi_n + \Omega(\alpha,\phi(\alpha)) - \pi_n)(g^y + g^{ind}) + (1+r_2)a_1N_1$$

$$= w_2N_2 + r_2s^*N_1 + (1+r_2)a_1N_1$$

$$- w^*N_1(1+\pi_n) - w^*N_1(\Omega(\alpha,\phi(\alpha)) - \pi_n)$$

$$+ \tau_w^*w^*N_1(1+\pi_n + \Omega(\alpha,\phi(\alpha)) - \pi_n)$$

$$- r^*s^*N_1 + r^*s^*N_1\tau_r - N_1(b^* + g^o + g^{ind})$$

$$- N_1(1+\pi_n + \Omega(\alpha,\phi(\alpha)) - \pi_n)(g^y + g^{ind})$$

$$= F(N_2, N_1(s^* + a_1)) - \delta N_1(s^* + a_1) + a_1N_1$$

$$- F(N(1+\pi_n), N_1s^*) + \delta N_1s^* - w^*N_1(\Omega(\alpha,\phi(\alpha)) - \pi_n)$$

$$+ \tau_w^*w^*N_1(\Omega(\alpha,\phi(\alpha)) - \pi_n) + \tau_w^*w^*N_1(1+\pi_n) + r^*s^*N_1t_r - N_1(b^* + g^o + g^{ind})$$

$$- N_1(1+\pi_n)(g^y + g^{ind}) - (\Omega(\alpha,\phi(\alpha)) - \pi_n)(g^y + g^{ind})$$

Note that at the initial steady state

$$\tau_w^*w^*(1+\pi_n) + r^*s^*\tau_r^* = (b^* + g^o + g^{ind}) + (1+\pi_n)(g^y + g^{ind})$$

$$
\begin{aligned}
SP_2 &= F(N_2, N_1(s^* + a_1)) - F(N(1+\pi_n), N_1 s^*) + (1-\delta)N_1 a_1 \\
&- w^*(1-\tau_w^*)N_1(\Omega(\alpha,\phi(\alpha)) - \pi_n) - N_1(\Omega(\alpha,\phi(\alpha)) - \pi_n)(g^y + g^{ind}) \\
&= F(N_2, N_1(s^* + a_1)) - F(N_2, N_1 s^*) + (1-\delta)N_1 a_1 \\
&+ F(N_2, N_1 s^*) - F(N_1(1+\pi_n), N_1 s^*) - \\
&(1-\tau_w^*)w^* N_1(\Omega(\alpha,\phi(\alpha)) - \pi_n) - N_1(\Omega(\alpha,\phi(\alpha)) - \pi_n)(g^y + g^{ind}) \\
&= F(N_2, N_1(s^* + a_1)) - F(N_2, N_1 s^*) + (1-\delta)N_1 a_1 \\
&+ N_1\{F(1+\Omega(\alpha,\phi(\alpha)), s^*) - F((1+\pi_n), s^*) \\
&- (1-\tau_w^*)w^*(\Omega(\alpha,\phi(\alpha)) - \pi_n) - (\Omega(\alpha,\phi(\alpha)) - \pi_n)(g^y + g^{ind})\} \\
&= N_1 \times \{F(1+\Omega(\alpha,\phi(\alpha)), s^* + a_1) - F(1+\Omega(\alpha,\phi(\alpha)), s^*) + (1-\delta)a_1\} \\
&+ N_1 \times \int_{1+\pi_n}^{1+\Omega(\alpha,\phi(\alpha))} [\frac{\partial F(z,s^*)}{\partial L} - (1-\tau_w^*)w^* - g^y + g^{ind}]dz \\
&= N_1 \int_{s^*}^{s^*+a_1} [\frac{\partial F(1+\Omega(\alpha,\phi(\alpha)), z)}{\partial K} + (1-\delta)]dz + N_1 \int_{1+\pi_n}^{1+\Omega(\alpha,\phi(\alpha))} [\frac{\partial F(z,s^*)}{\partial L} - c^{y*} - s^* - g^y - g^{ind}]dz
\end{aligned}
$$

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Figure 1

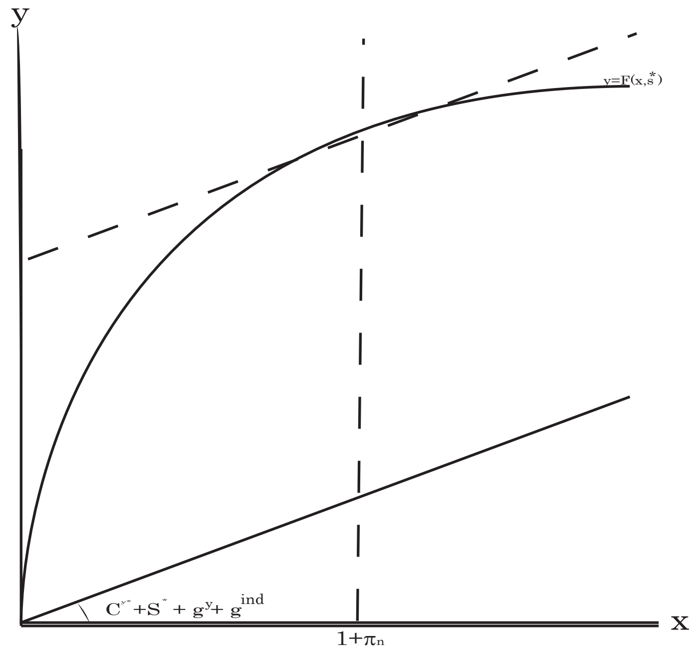

Figure 2

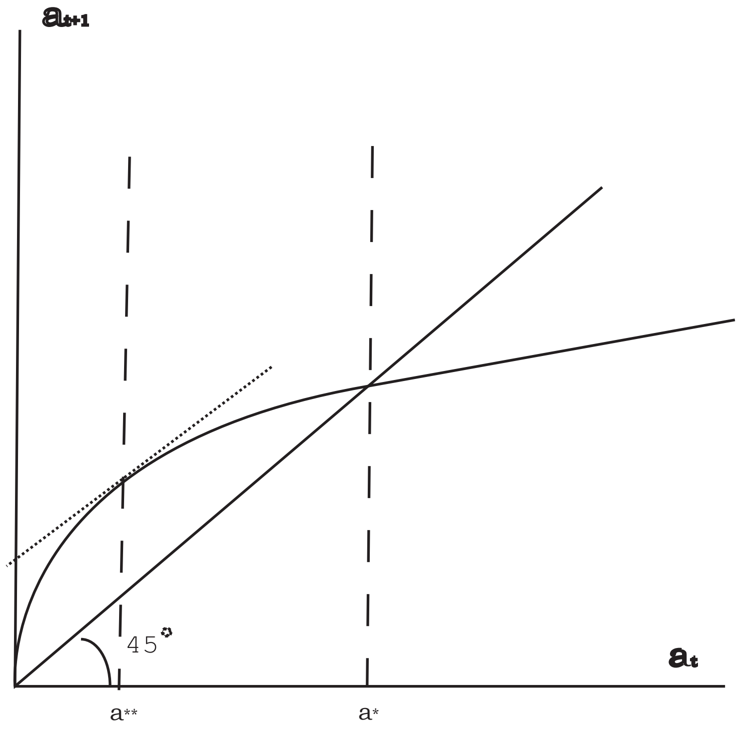

Figure 3

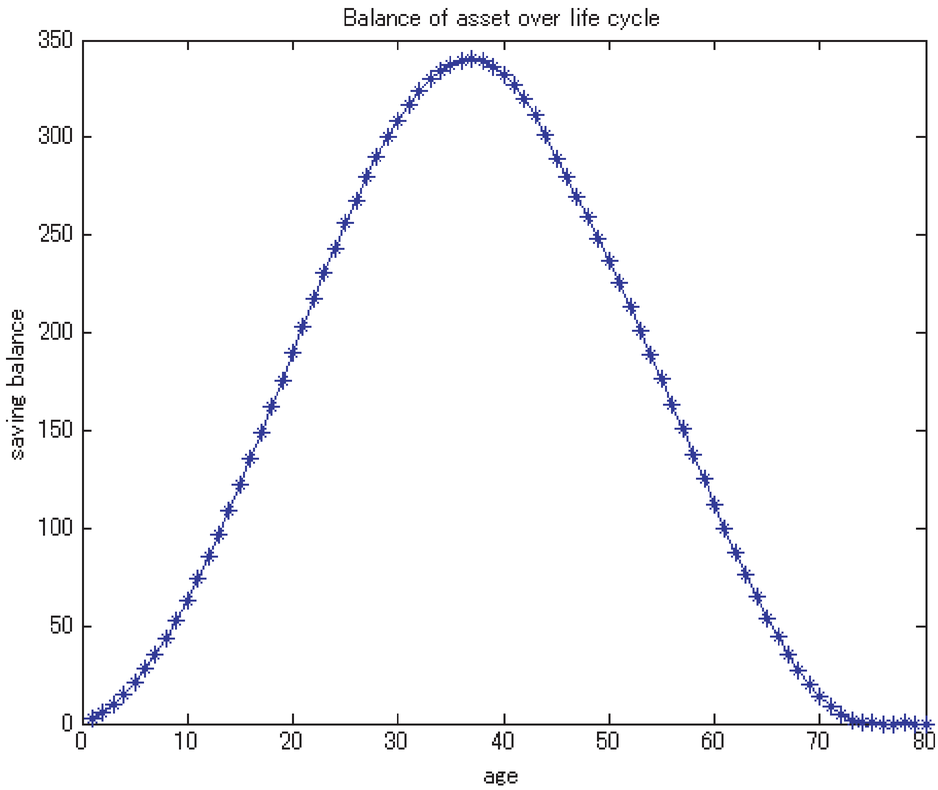

Figure 4

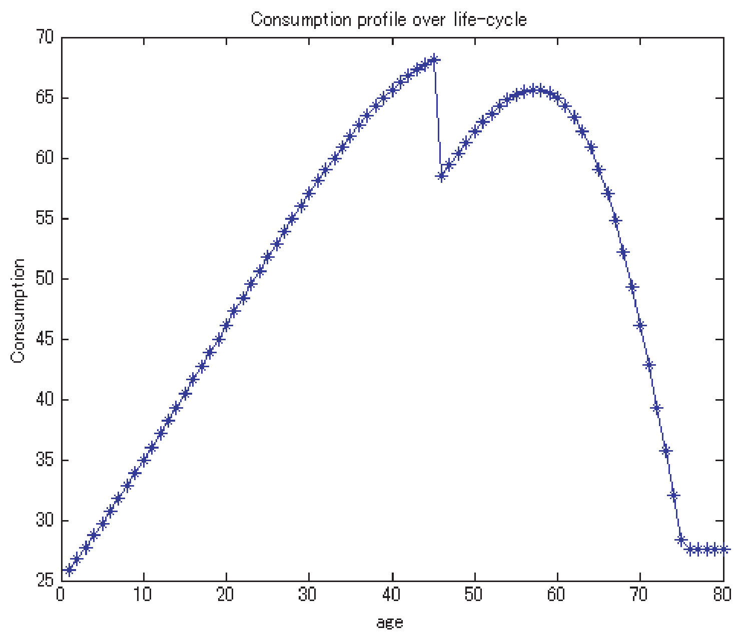

Figure 5

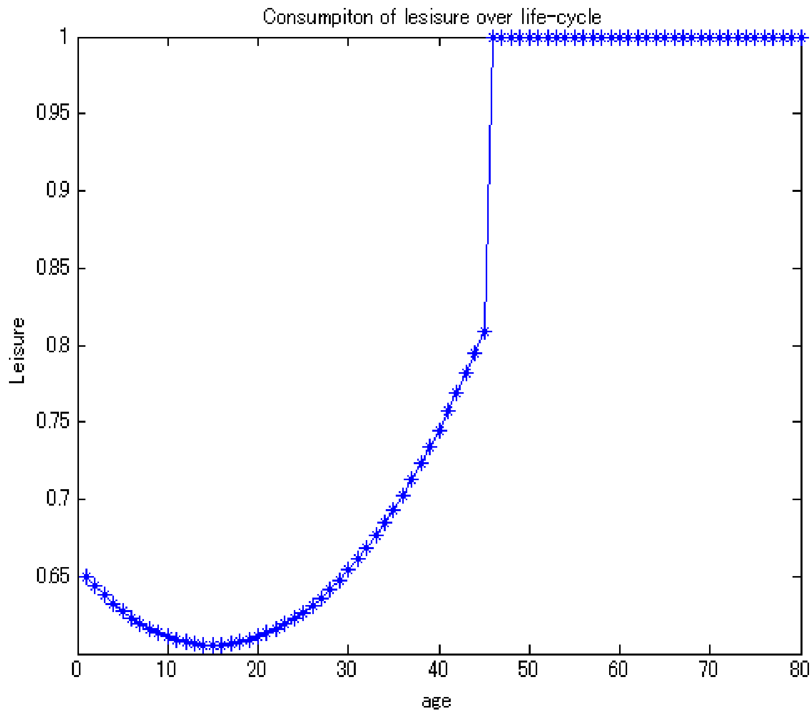

Figure 6

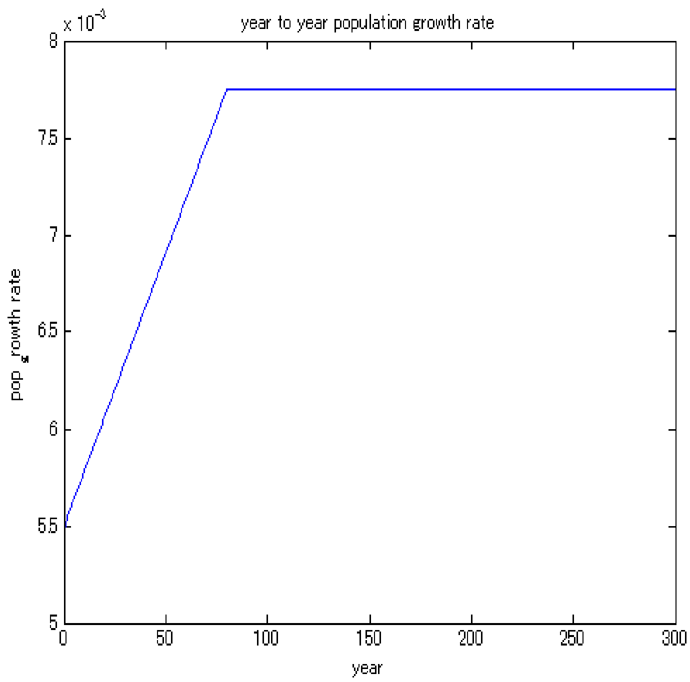

Figure 7

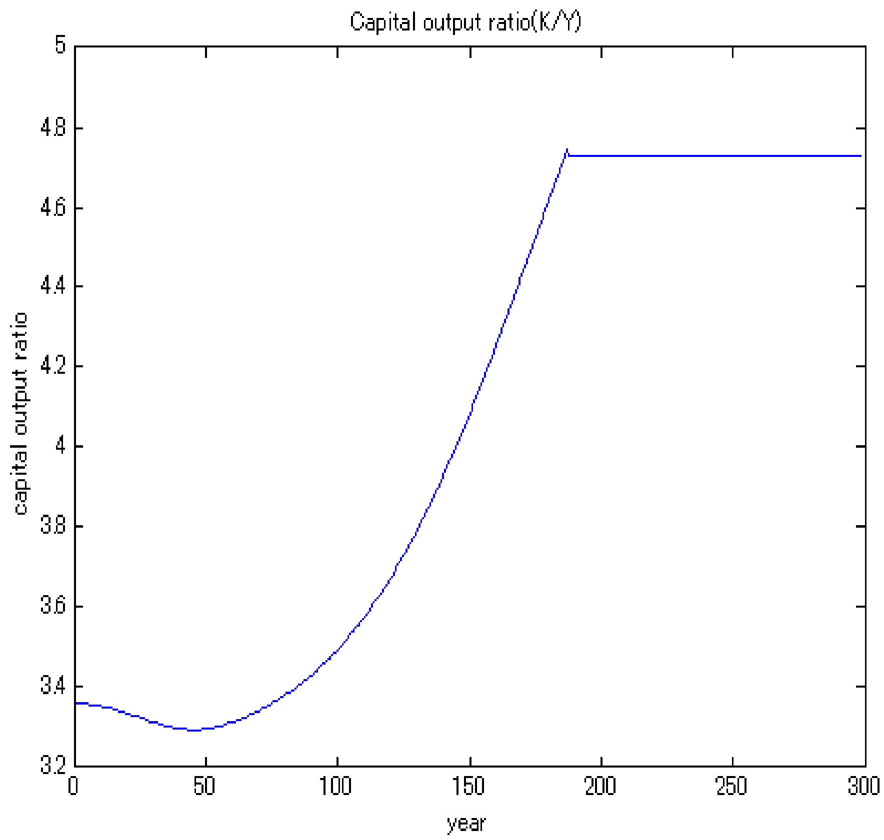

Figure 8

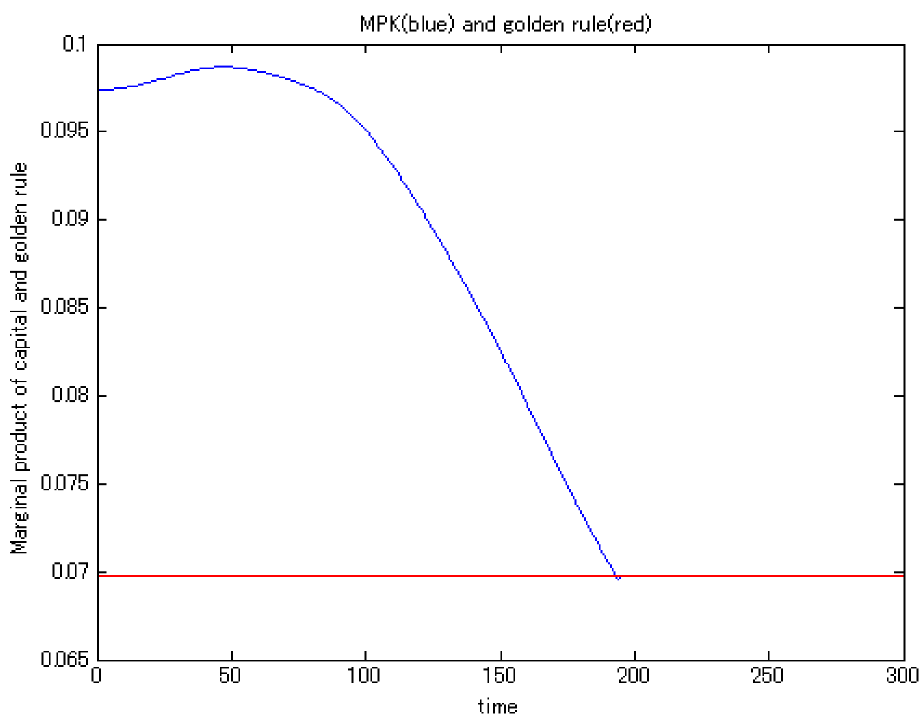

Figure 9

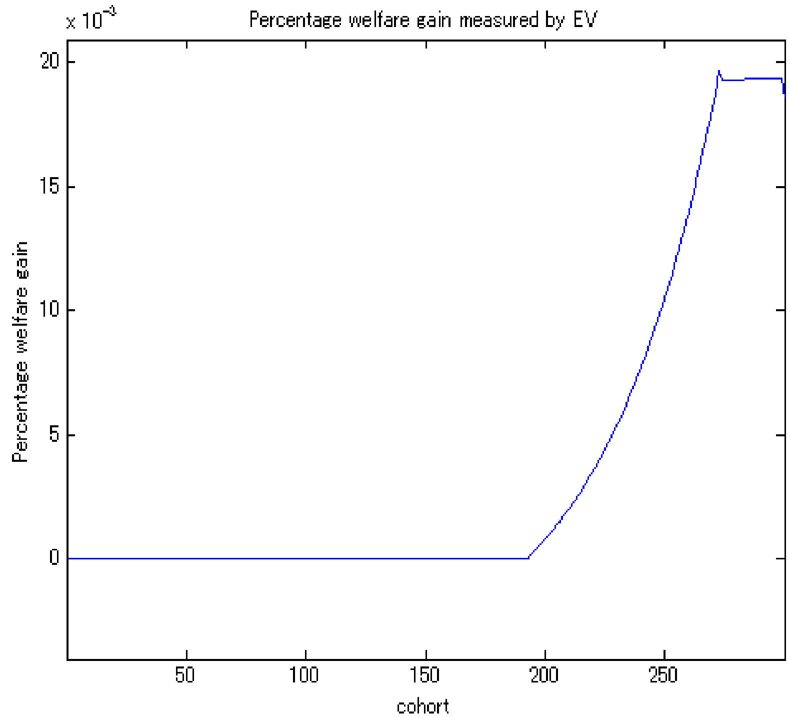

Figure 10

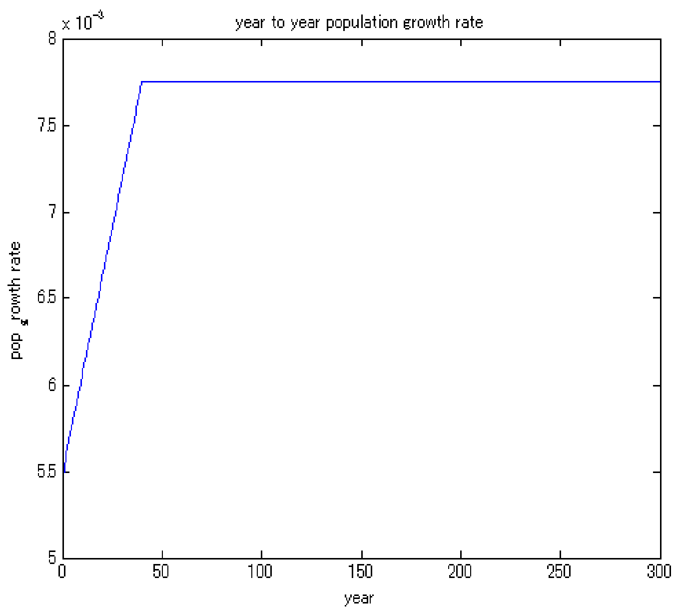

Table 1: The welfare effect of Immigration

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Case No.	Initial population growth rate	targent population growth rate	years taken to reach the target population growth rate	CRRA	distributio n share of the net surplus	replacement ratio	years to reach goldren rule	% increase of capital stock per capita at a new s.s	%change of publicly provided private goods per capita at new s.s.	% change of welfare of the cohort born at a new s.s.	% change of the sum of present discount value of welfare
					Bench mark case :distributional share =100%						
1	0.55%	0.775%	81	3	100%	0.6	187	66.67%	19.04%	2.01%	0.014%
2	0.55%	1.00%	81	3	100%	0.6	163	58.95%	19.52%	2.05%	0.027%
3	0.55%	1.450%	81	3	100%	0.6	144	45.20%	19.87%	2.10%	0.042%
4	0.55%	1.900%	81	3	100%	0.6	141	33.33%	19.40%	2.05%	0.044%
					distribtional share= 90%						
5	0.55%	0.775%	81	3	90%	0.6	216	66.67%	19.04%	2.01%	0.020%
6	0.55%	1.00%	81	3	90%	0.6	187	58.95%	19.52%	2.06%	0.035%
7	0.55%	1.450%	81	3	90%	0.6	163	45.20%	19.87%	2.10%	0.051%
8	0.55%	1.900%	81	3	90%	0.6	157	33.33%	19.40%	2.05%	0.052%
					distributional share =80%						
9	0.55%	0.775%	81	3	80%	0.6	264	66.67%	19.04%	2.01%	0.029%
10	0.55%	1.00%	81	3	80%	0.6	227	58.95%	19.52%	2.06%	0.046%
11	0.55%	1.450%	81	3	80%	0.6	196	45.63%	19.87%	2.10%	0.065%
12	0.55%	1.900%	81	3	80%	0.6	188	33.33%	19.40%	2.05%	0.066%
					distributional share =70%						
13	0.55%	0.775%	81	3	70%	0.6	363	66.67%	19.04%	2.01%	0.033%
14	0.55%	1.00%	81	3	70%	0.6	313	58.95%	19.52%	2.06%	0.053%
15	0.55%	1.450%	81	3	70%	0.6	274	45.37%	19.87%	2.10%	0.074%
16	0.55%	1.900%	81	3	70%	0.6	271	33.46%	19.40%	2.05%	0.076%
					distributional share =60%						
17	0.55%	0.775%	81	3	60%	0.6	*	52.11%	16.78%	1.75%	0.030%
18	0.55%	1.00%	81	3	60%	0.6	*	48.05%	18.38%	1.88%	0.056%
19	0.55%	1.450%	81	3	60%	0.6	*	35.42%	19.22%	2.00%	0.081%
20	0.55%	1.900%	81	3	60%	0.6	*	21.90%	18.70%	1.96%	0.084%

Notes

1. * mark means that the economy's capital labor ratio keep increasing even after 500 years and it does not reach the goden rule level after 500 years. Thus, for computational reason, the increased government expenditure and the increased capital stock per capita is calculated as if the economy reach the steady state at 500 years.

2. The wage tax rate at the initial steady state is 20.9 percent.

Table 2: The welfare effect of immigrants with higher speed of acceptance

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Case No.	Initial population growth rate	targent population growth rate	years taken to reach the target population growth rate	CRRA	distribution share of the net surplus	replacement ratio	years to reach goldren rule	% increase of capital stock per capita at new s.s.	%change of publicly provided private goods per capita at new s.s.	% change of welfare of the cohort born at the new s.s.	% change of the sum of present discount value of welfare
distributional share=100%											
1	0.55%	0.775%	40	3	100%	0.6	195	63.37%	17.57%	1.87%	0.013%
2	0.55%	1.00%	40	3	100%	0.6	173	55.81%	17.84%	1.89%	0.023%
3	0.55%	1.450%	40	3	100%	0.6	161	42.33%	17.81%	1.89%	0.030%
4	0.55%	1.900%	40	3	100%	0.6	183	30.69%	16.98%	1.81%	0.017%
distributional share=90%											
5	0.55%	0.775%	40	3	90%	0.6	226	63.37%	17.57%	1.87%	0.024%
6	0.55%	1.00%	40	3	90%	0.6	198	55.81%	17.85%	1.90%	0.036%
7	0.55%	1.450%	40	3	90%	0.6	180	42.33%	17.81%	1.90%	0.043%
8	0.55%	1.900%	40	3	90%	0.6	189	30.69%	16.98%	1.81%	0.029%
distributional share=80%											
9	0.55%	0.775%	40	3	80%	0.6	278	63.37%	17.57%	1.87%	0.029%
10	0.55%	1.00%	40	3	80%	0.6	243	55.81%	17.85%	1.90%	0.043%
11	0.55%	1.450%	40	3	80%	0.6	194	42.33%	17.81%	1.90%	0.052%
12	0.55%	1.900%	40	3	80%	0.6	223	30.69%	16.98%	1.81%	0.038%
distributional share=70%											
13	0.55%	0.775%	40	3	70%	0.6	393	63.37%	17.58%	1.87%	0.033%
14	0.55%	1.00%	40	3	70%	0.6	345	55.81%	17.85%	1.90%	0.050%
15	0.55%	1.450%	40	3	70%	0.6	318	42.33%	17.81%	1.90%	0.062%
16	0.55%	1.900%	40	3	70%	0.6	348	30.69%	16.98%	1.81%	0.048%
distributional share=60%											
17	0.55%	0.775%	40	3	60%	0.6	*	51.49%	16.99%	1.69%	0.037%
18	0.55%	1.00%	40	3	60%	0.6	*	48.28%	18.43%	1.89%	0.061%
19	0.55%	1.450%	40	3	60%	0.6	*	35.49%	19.23%	2.00%	0.086%
20	0.55%	1.900%	40	3	60%	0.6	*	21.55%	18.59%	1.94%	0.058%

Notes

1. * mark means that the economy's capital labor ratio keep increasing even after 500 years and it does not reach the goden rule level after 500 years. Thus, for computational reason, the increased government expenditure and the increased capital stock per capita is calculated as if the economy reach the steady state at 500 years.

2. The wage tax rate at the initial steady state is 20.9 percent.

Table 3: The welfare effect of Immigration with different CRRA and replacement rate

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Case No.	Initial population growth rate	targent population growth rate	years taken to reach the target population growth rate	CRRA	distribution share of the net surplus	replacement ratio	years to reach goldren rule	% increase of capital stock per capita at new s.s.	%change of publicly provided private goods per capita at new s.s.	% change of welfare of the cohort born at the new s.s.	% change of the sum of present discount value of welfare
					CRRA=2						
1	0.55%	0.775%	81	2	100%	0.6	199	36.90%	9.06%	1.07%	0.050%
2	0.55%	1.00%	81	2	100%	0.6	158	30.56%	10.09%	1.19%	0.098%
3	0.55%	1.450%	81	2	100%	0.6	123	19.27%	11.54%	1.36%	0.177%
4	0.55%	1.900%	81	2	100%	0.6	108	9.52%	12.17%	1.44%	0.226%
					CRRA=4						
5	0.55%	0.775%	81	4	100%	0.6	181	97.19%	30.99%	3.00%	0.005%
6	0.55%	1.00%	81	4	100%	0.6	169	88.06%	31.12%	3.02%	0.007%
7	0.55%	1.450%	81	4	100%	0.6	184	71.19%	30.73%	2.99%	0.004%
8	0.55%	1.900%	81	4	100%	0.6	**	**	**	**	**
					Replacement rate=0.55						
9	0.55%	0.775%	81	3	100%	0.55	200	63.37%	17.57%	1.87%	0.011%
10	0.55%	1.00%	81	3	100%	0.55	176	55.81%	17.85%	1.90%	0.021%
11	0.55%	1.450%	81	3	100%	0.55	161	42.33%	17.81%	1.89%	0.030%
12	0.55%	1.900%	81	3	100%	0.55	170	30.69%	16.98%	1.81%	0.024%
					Replacement rate=0.5						
13	0.55%	0.775%	81	3	100%	0.5	217	60.05%	16.12%	1.73%	0.007%
14	0.55%	1.00%	81	3	100%	0.5	195	52.64%	19.19%	1.74%	0.014%
15	0.55%	1.450%	81	3	100%	0.5	190	39.43%	15.77%	1.69%	0.016%
16	0.55%	1.900%	81	3	100%	0.5	**	21.90%	18.70%	1.96%	0.08%

Note
** mark means that it is not feasible to accept immigrants in a Pareto-improving way with this size and speed of accepting immigration