Pareto-improving Immigration in the Presence of the Social Security

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Abstract

This paper qualitatively and quantitatively analyzes the welfare effect of accepting immigrants in the presence of a pay-as-you-go social security system. First, it demonstrates that if there are intergenerational government transfers from the young to the old in the sense that the marginal product of labor of a young individual is greater than what he or she receives, including publicly provided private good, accepting immigrants Pareto-improves welfare. Second, the paper shows that if there are intergenerational government transfers in the sense defined above, the government can achieve a path that leads to the golden rule level of capital stock per capita within a finite time in a Pareto-improving way by accepting immigrants and by adjusting the wage tax and the capital income tax. Third, this paper presents how those taxes should be adjusted when immigrants are accepted. Fourth, using the computational overlapping generation model developed by Auerbach and Kotlikoff (1987), I simulate the model economy and calculate years needed to reach the golden rule level of capital stock per capita and the present discounted value (PDV) of the Pareto-improving welfare gain obtained by accepting immigrants. In this simulation, I consider a moderate increase of immigrant/native ratio(INR) where INR starts to increases from 15.5 % and reaches 25.5 % at 100th year and it remains at 25.5 % all later years. My simulation shows that (1) in all cases considered, all cohorts are Pareto-improved,(2) as the share of the surplus for the government saving becomes higher, the economy reaches the golden rule earlier, (3) if the share of the surplus for the government saving is greater than or equal to 70 percent, the economy reaches the golden rule level between 100 and 200 years in a Pareto-improving way, (4) the PDV of the Pareto-improving welfare gain amounts to 20 to 30 percent of the initial GDP. In addition, I conducted robustness checks of my results by changing the values of several parameters such as the replacement rate, the initial government debt level, immigrant earning level and preference parameters. In those robustness checks, I find that the magnitude of the welfare gain does not change for the reasonable ranges of parameter values. All results(both theoretical and computational) indicate that the economic effect of accepting immigrants is not trivial.

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1 Introduction

Accepting a constant flow of immigrants brings a higher population growth rate to a host country. With a neoclassical production function that exhibits the diminishing marginal product of capital, the standard growth model (Solow (1964)) predicts that such a higher population growth rate leads to both lower levels of consumption per capita and income per capita, starting from a dynamically efficient initial steady state. On the other hand, recently there are increasing interests among policy makers in accepting immigrants as a policy response to the negative demographic shocks in the presence of a pay-as-you-go social security system. In addition, in the public finance literature, there is an increasing interest in the effect of immigration on the social welfare in a dynamic general equilibrium model By using the computational overlapping generation model (Auerbach and Kotlikoff model(Auerbach and Kotlikoff(1998)), Storesletten(2000) demonstrated that accepting a particular type of immigrants, (skilled old aged immigrants who will not be able to claim the social security benefit due to the requirement of the minimum duration of the social security tax payment) will increase the social welfare in the presence the retirement of the baby boom generation. On the other hand, Fehr, Jokisch and Kotlikoff (2004) argued that such a welfare gain does not exist. Feldstein (2006) analyzed the effect of immigration in Spain and concluded that immigration does not bring welfare gain.

Given the mixed results in the literature regarding the effect of accepting immigrants on the social welfare, a natural question arises whether accepting immigrants Pareto-improves welfare or not from the theoretical point.

This article analyzes such a question. Firstly, using the overlapping generation model developed by Diamond(1965), I present a sufficient condition under which accepting immigrants can Pareto-improve welfare. Secondly, I demonstrate that accepting immigrants and implementing a relatively simple compensating tax policy can achieve this Pareto-improvement. Thirdly, I analytically show that the government can design immigration and tax policy that lead the economy to the golden rule level of capital stock per capita within a finite time in a Pareto-improving way. Finally, I quantity the welfare effect of accepting immigrants on the economy in the presence of a pay-as-you-go social security system by using the computation overlapping generation model (Auerbach and Kotlikoff model). I consider a moderate increase of immigrant/native ratio (INR) where INR starts to increase from 15.5 percent, reaches 25.5 at 100th years and remains 25.5 percent in later years. With this speed of increase of INR and in the model that mimics important dimensions of the US economy, my simulation shows it takes 111 years for the model economy to reach the golden rule with (weak) Pareto-improvement of all cohorts. At the new balanced growth path, the capital stock per efficient unit of labor increases by 115 percent and the publicly provided private goods per capita increases by 53 percent. The PDV of the Pareto-improved utilities, measured by the expenditure function, of natives and their descendant, which does not include the immigrant and their descendants, amount to 20 percent of initial GDP. When the time to reach the target INR is shorten to 50 years, the PDV of the Pareto-improvement amounts to 31 percent of the initial GDP. Finally, for the robustness checks, I conducted simulations by using different share of the surplus for the government saving, different replacement rate, different time preference rate, different risk aversion, different initial government debt(asset) levels ,different immigrant earning level and different INR. Those simulations show that my simulation results do no change substantially in magnitude for different parameter values.

The organization of this paper is as follows. In section 2, I present a brief limited literature review. In section 3, I present a theoretical analysis. In section 4, I present a simulation-based analysis using the computation overlapping generation model. In section 5, I present a conclusion.

2 Literature

Auerbach and Kotlikoff (1999), Miller (1999) and Storesletten (2003) are the first group of papers that analyzed the welfare effect of accepting immigrants in a dynamic computational general equilibrium model. Storesletten showed that if the government selectively accepts immigrants, then it is possible to soften the negative shocks due to the retirement of the baby boom generation. On the other hand, using a more detailed model, Fehr, Jokisch and Kotlikoff (2004) argued that the welfare gain of accepting immigrants is almost zero or very small if it is positive. Feldstein (2006) analyzed the fiscal effect of accepting immigrants in Spain. He argued that accepting immigrant will not be likely to the solution since in later periods, the government needs to pay the social security benefit of the retired immigrants.

In the theoretical analysis, in a series of papers, Razin (1999) is the first paper that showed that accepting immigrant improves welfare. He showed that in a small open economy model where factor prices are fixed, accepting immigrant can improve welfare in the presence of PYGO social security. However, in the subsequent paper, Razin and Sadka (2000) showed that in an closed economy model where capital accumulation is endogenous, the result obtained under the assumption of a small open economy is not likely to hold and accepting immigrant will worse the welfare of the native. Their conclusion led Razin, Sadka and Suwankiri(2011) to the analysis of political economy models where there is a conflict among different agents regarding accepting immigrant.[1]

In the literature of the effect of the immigration on the host country, several researchers conducted the cost benefit analysis of accepting immigrants (For example,Huddle(1993), Borjas(1994), Passel (1994). Simon (1984) Akbari(1989), Canova and Ravn (1998), Auerbach and Preopoulos (1999), and Storesletten(2003)). In many of studies the inclusion of the social security benefit to the retired immigrants is an important factor to determine whether accepting immigrants is beneficial to the host country. However, whether the social security benefit to the retired immigrants should be included as the cost of accepting immigrants is not clear. Under the pay-as-you-go social security system, the social security benefit of the old at the time t is paid by the young at the time t. If a researcher includes the social security benefit of the retired immigrants as a fiscal cost, then the tax paid by the children of the immigrants should be

[1] In the theoretical analysis, there is another strand of ligature which analyzed the optimal population growth in the growth model. A starting paper in this literature is Samuelson (1975) and he analyzed the welfare maximizing optimal population growth rate in the standard neoclassical growth model. On the other hand, the subsequent analysis proved that the optimal population growth rate in many case does not maximize the welfare and that it rather minimizes the welfare. (Deardorfff 1976). The reason that the first order condition of the optimal population growth rate is not optimal is the indirect utility function becomes quasi-convex with respect to the population growth rate and the consumption. Michel and Pestieau identified the parameter values where the first order condition of the optimal population growth rate is actually optimal. Those studies suggest that if want to find an optimal population growth rate, we need to be careful on the sufficient condition. Due to the concern of the non-optimality of the first order condition, in this paper I do not look for the parameter value of the optimal population growth rate or optimal immigration rate. Instead, I check whether a small change of the population growth rate will Pareto-improve welfare or not.

included as a fiscal benefit. However, in the pay-as-you-go social security system, the payment of the social security tax and the benefit to the retired are approximately the same. This implies that the inclusion of the social security benefit to the retired immigrants as the fiscal cost might not necessary.

My paper contributes to the existing literature in several ways. First, I show that even in the closed economy it is Pareto-improving to accept immigrants in the presence of the social security if the marginal product of labor of the young is greater than resource allocated to young, which is equal to the sum of the after-tax income and publicly provided private good of the young (MPL condition). Note that the resource allocated to the young does not include the social security benefit that they will receive when the young becomes old. Thus, in the presence of PYGO social security tax and benefit, this MPL condition is likely to be satisfied.This is contrary to Razin and Sadka(2000), but is consistent with Razin(1999) and extends his result to the closed economy. Second, I show that if this MPL condition is satisfied, the government can make the economy reach the gold-rule level in Pareto-improving way within a finite time by accepting immigrants. Third, I show that accepting immigrant and a simple tax adjustment achieve the Pareto-improvement and transition to the golden rule. Fourth, using 80 period computational overlapping generation model developed by Auerbach and Kotlikoff(1987), I show that the model economy is actually Pareto-improved and reaches the golden rule level by accepting immigrants in a reasonable way. I show how long it takes for the economy to reach the gold rule level and calculate the welfare gain of Pareto-improvement when the size of immigrant is increased in a reasonable size in the model economy. In addition, I simulate the model using different values of parameters and conduct robustness checks.

3 Analysis

3.1 The model

The model uses the standard overlapping generation model with a neoclassical production function developed by Diamond (1965). At every period, a continuum of individuals is born. Individuals who are born at period t is defined as cohort t. Each cohort lives for two periods. When individuals are in the first period, they work and they are called "young". When they are in the second period, they are retired and they are called "old". I assume that immigrants come only when they are young and that the government at the host country prohibits the immigrants to immigrate when they are already old. Each cohort of the native and the immigrants supplies one unit of labor inelastically when they are young[2]. Let j be the index indicating a place of birth. If an individual is a native, $j = n$ and she/he is an immigrant, $j = m$. Let N_t^j be the number of the young of type j at the period t. Let $(c_t^{y,j}\ c_{t+1}^{o,j})$ be the consumption at the young period and the old period of type j $(j = n, m)$ agent who is born at the period t. Let g_t^y be the amount of publicly provided private goods for the native or immigrant for the young native or immigrant that is consumed at period t such as education and government provided health care service for the young. Let g_t^o be the amount of publicly provided private goods for the old native or immigrant at the period t such as medicaid and publicly provided nursing home. Let g_t^{ind} be the amount of age-independent publicly provided private goods such as police service. [3] I assume that the utility function of cohort of the native is

$$U(c_t^{y,n}, c_{t+1}^{o,n}, g_t^y, g_{t+1}^o, g_t^{ind}, g_{t+1}^{ind}) = u^y(c_t^{y,n}) + v^y(g_t^y, g_t^{ind}) + \frac{1}{1+\rho}[u^o(c_t^{o,n}) + v^o(g_{t+1}^o, g_{t+1}^{ind})] \tag{1}$$

I assume that $u^i(c_t^{i,n})$ and $v^i(g_t^i, g_t^{ind})$ $(i = y, o)$ are are strictly increasing and concave function. I assume additive separability of publicly provided private goods so that the provision of publicly provided private goods does not affect the consumption and saving decision of consumers. This assumption simplifies the analysis since I assume that the government redistributes the welfare gain of accepting immigrants in the form of increased publicly provided private goods to consumers to avoid the income effect.[4] I assume that publicly provided private goods are supplied independent of nationality. Thus, $(g_t^y, g_t^o, g_t^{ind})$ does not have

[2] In section 3.3, I show that the main results of this paper hold even when the labor supply is elastic. I use the assumption of the inelastic labor supply to simplify the analysis in this section.

[3] In this paper, I ignore non-rivalry public goods. Note that the presence of non-rivalry public goods will favor immigration since accepting immigration means that the cost of non-rivalry public goods will be shared by more individuals without decreasing the consumption of non-rivalry public goods.

[4] There are several ways to redistribute the welfare gain to consumers. I use this method to simplify the analysis. Using other ways for redistributing the welfare gain, the main conclusion does not change.

superscript j. The assumption of the above utility function also implies that labor supply is inelastic. In the section 3.3, I will relax this assumption and show that the results in the section 2 hold even if the labor supply is elastic. Let s_t^j be the saving of the nationality of j who are born or immigrate at the period t. Let w_t^i and τ_{wt}^jbe the pre-tax wage rate and the wage tax rate of nationality j who are born or immigrate at the period t. Let r_t and τ_{rt} be the pre-tax interest rate and the tax rate on interest at period t. Let b_t^j be the social security benefit that is given to the old of type j at period t. Cohort t maximizes the above utility function subject to the budget constraint. The budget constraint of the cohort t of type j is

$$w_t^j(1-\tau_{wt}) = c_t^{y,j} + s_t^j \text{ and } b_{t+1}^j + (1+(1-\tau_{rt})r_t)s_t^j = c_{t+1}^{o,j} \tag{2}$$

For the production side, let $F(L_t, K_t)$ be a production function where L_t and K_t are the total capital stock and the total amount of labor used at period t, respectively. I assume that $F(L_t, K_t)$ exhibits constant returns to scale and that both the marginal product of labor and capital are diminishing. Let δ be the depreciation rate of the capital. I assume that the Inada condition is satisfied:

$$F(x,0)=0,\ F(0,x)=0,\ \lim_{k+\to 0}\frac{\partial F(x,k)}{\partial K}\to\infty,\quad \lim_{k\to\infty}\frac{\partial F(x,k)}{\partial K}\to 0,\ \lim_{l+\to 0}\frac{\partial F(l,x)}{\partial L}\to\infty \ \text{ for any } x>0. \tag{3}$$

One objective of this paper is to analyze whether accepting immigrants can Pareto-improve welfare or not. In the literature, it is well-known that if the market interest rate is lower than the population growth rate, it is possible to Pareto-improve welfare (dynamic inefficiency). Since this paper's interest is not such a dynamic inefficiency problem, I postulate that at the initial steady state, the market interest rate is higher than the population growth rate (Cass(1972)). Furthermore, for the welfare analysis of immigration, I make the following additional assumptions.

AS1: The economy is at the steady state before the government accepts immigrants.

AS2: The amount of publicly provided private goods, $(g_t^y, g_t^o, g_t^{ind})$ per person is constant at the initial steady state and $(g_t^y, g_t^o, g_t^{ind}) = (g^y, g^o, g^{ind})$..

AS3: The government uses a pay-as-you-go social security system at the initial steady state.

AS4: Immigrants and the native must be treated equally in the tax system once the immigrants are accepted in the host country.

AS5: For the treatment of immigrants regarding the social security system, the government collects the social security tax from the immigrants in the same way as it collects from the native and it pays the benefit in the same way as to the native

AS6: Immigrants has $\phi \times 100$ percent efficient unit of labor as the native.

AS7: Immigrants and the native has the same preference.

AS8: The descendant of immigrants assimilates to the native and earns the same income as the native.

AS9: Immigrants and their children stay permanently in the host country.

AS10: The fertility rate of immigrants is equal to or higher than the fertility of the native.

AS11: At the initial steady state, the government does not impose a distorting tax such as capital income tax.

AS1 is standard for the policy analysis where the model involves a dynamic dimension. I use the AS2 to focus on the issue of immigration rather than issues on public expenditure. AS3 comes from the fact that the social security of most countries is the pay-as-you-go system. AS 4 and AS 5 need more discussion. Clearly, if the government can treat the immigrants discriminatory in the tax and the public pension systems, there is a way to increase the utility of both natives and the immigrants. Since the wage rate of the immigrants in their home country is lower than the wage rate in the host country, if the government in the host country can set a high tax rate on incoming immigrants in a way that the net wage rate of the immigrants in the host country is still higher than the wage rate of the immigrants in the home country and if the government redistributes the tax revenue collected from the immigrants to the native, it is possible to Pareto-improve welfare of both the native and the immigrants. AS4 and AS5 delete such an obvious case. In addition, AS4 can be justified from a political reason. Although imposing a high tax rate on the incoming immigrants and redistributing the tax revenue to the native can be Pareto-improving for both the native and the immigrants, such a discriminatory policy might be sometimes illegal from a constitutional point. Also such a discriminatory policy is not feasible when the native does not want to see the immigrants treated in a discriminatory way. Thus, in the following analysis, I assume that in the tax and the social security policy, the government must treat the immigrants and the natives in the same way once immigrants are accepted. Even under such a constraint, I will show that it is possible to Pareto-improve welfare by accepting immigrants if a certain condition is met. The AS6 and AS7 are not critical for proposition 1. AS8, AS9 and AS10 are used to simplify the analysis. I use AS11 because of our interest in the welfare effect of immigration. If there is already distorting tax at the initial steady state in the economy, clearly the government can Pareto-improve the initial steady state by correcting those taxes. Because the labor supply is inelastic, the wage tax is optimal and the use of capital income tax is sub-optimal. Since the focus of this paper is the first order effect of immigration, not the tax reform, I use the AS11. Of course, in reality, the governments of the most developed countries impose capital income taxes. In the section 3.3, I show that the main conclusion does not change even in the presence of capital income taxation.

Now consider the government budget constraint at period t. At this point, consider the government budget constraint with the capital income tax although in this section it is assumed that the capital income tax is not used at the initial steady state. The government budget constraint at period t is

$$\tau_{wt+1} \sum_{j=n,m} w_{t+1}^j N_{t+1}^j + \tau_{rt+1} r_{t+1} \sum_{j=n,m} s_t^j N_t^j - (b_t + g^o + g^{ind} - (1+r_t) a_t) \sum_{j=n,m} N_t^j - (g^y + g^{ind} - a_{t+1}) \sum_{j=n,m} N_{t+1}^j = 0. \tag{4}$$

where a_t is the government saving (debt) balance per capita at period t. b_t is the social security benefit at the period t.

Then, w_{t+1}^j and r_{t+1} are determined as follows:

$$w_{t+1}^n = \frac{\partial F(N_{t+1}^n + \phi N_t^m, \sum s_t^j N_t^i + a_t \sum N_t^j)}{\partial L}, \ w_{t+1}^m = \phi \frac{\partial F(N_{t+1}^n + \phi N_t^m, \sum s_t^j N_t^j + a_t \sum N_t^j)}{\partial L}$$
$$\text{and} \quad r_{t+1} = \frac{\partial F(N_{t+1}^n + \phi N_t^m, \sum s_t^j N_t^j + a_t \sum N_t^j)}{\partial K} - \delta \tag{5}$$

From the individual budget constraint, $w_{t+1}^j(1 - \tau_{wt+1}) = c_{t+1}^{y,j} + s_{t+1}^j$ and $c_{t+1}^{o,j} = (1 + r_{t+1}(1 - \tau_{rt+1}))s_t^j +$

b_{t+1}. Solving for $\tau_{wt+1}w^j_{t+1}$ and $\tau_{rt+1}r_{t+1}s^j_t$ and substituting them into the government budget constraint, we have the following resource constraint:

$$F(N^n_{t+1} + \phi N^m_t, \sum s^j_t N^j_t + a_t \sum N^j_t) + (1-\delta)\{\sum s^j_t N^j_t + a_t \sum N^j_t\}$$
$$= \sum\{c^{y,j}_{t+1} + s^j_{t+1} + g^y + g^{ind}\}N^j_{t+1} + a_{t+1}\sum N^j_{t+1} + \sum N^j_t\{c^{o,j}_{t+1} + g^o + g^{ind}\} \quad (6)$$

Let the fertility rate of the native and the immigrant be π_n and π_m. I assume that $1+\pi_n > 0$ and $1+\pi_m > 0$. When $1+\pi_j \leq 0$ for $j = n, m$, the population of type j becomes less or equal to zero immediately. Given π_n and π_m, N^n_t can be written as follows:

$$N^n_t = (1+\pi_m) \times N^m_{t-1} + (1+\pi_n) \times N^n_{t-1} \quad (7)$$

Let α_t be the ratio of immigrant to the native. The immigration policy is expressed in terms of α_t. For example, one time acceptance of immigration means that $\alpha_0 = 0$, $\alpha_1 = \alpha$ and $\alpha_t = 0$ for $t = 2, 3, ...$ Permanently accepting immigrants means that $\alpha_0 = 0$ and $\alpha_t = \alpha$ for all $t = 1, 2, , , ,$. The population growth rate of the native is

$$\frac{N^n_{t+1}}{N^n_t} - 1 = (1+\pi_m) \times \alpha_t + (1+\pi_n) - 1$$
$$= (1+\pi_m) \times \alpha_t + \pi_n \quad (8)$$

Note that $N^n_t + N^m_t$ be the total population who is born or immigrate at the period t. the growth rate of the total population is

$$\frac{(1+\alpha_{t+1})\{(1+\pi_m) \times \alpha_t + (1+\pi_n)\}}{(1+\alpha_t)} - 1$$

3.1.1 The effect of immigration through inter-generational channel

When immigrants are accepted, they affect the welfare of the native in two channels. The first one is the inter-generational channel. The immigrant will bring an inflow of labor and an inflow of labor offsets the effect of the pre-existing PYGO social security. This affects capital labor ratio and might be able to enhance the efficiency of the economy. The second channel is the intra-redistribution effect. When the average earning of immigrants is lower than the average earning of the natives but an immigrant consumes the same amount of publicly provided private goods such as public education, welfare program and health services as an native on average, the inflow of immigrants imposes fiscal burden to the government budget and this tighter government budget is likely to reduce the welfare of the native. First, this section focuses on the inter-generational effect and its long run implication on capital accumulation. To see the inter-generational effect in a transparent way, I assume that the efficiency unit of labor of incoming immigrant is the same as the native and that there is no intra-redistribution channel in the following analysis. This implies that $\phi = 1$ in our model. The case that includes both inter-generational effect and intra-redistribution effect is analyzed in the section 3.5.

Using the assumption that the native and immigrants have the same preference and $\phi = 1$, the resource constraint becomes as follows:

$$F(l^n\{(1+\pi_m)\times\alpha_t+(1+\pi_n)\}\times\frac{(1+\alpha_{t+1})}{(1+\alpha_t)}, s_t^n+a_t)+(1-\delta)(s_t^n+a_t)\geq$$
$$\{c_{t+1}^{y,n}+s_{t+1}^n+g^y+g^{ind}+a_{t+1}\}\{1+\pi_m)\times\alpha_t+(1+\pi_n)\}\times\frac{(1+\alpha_{t+1})}{(1+\alpha_t)}+\{c_{t+1}^{o,n}+g^o+g^{ind}\}$$

Now consider the individual intertemporal optimization problem. The individual intertemporal first order condition is

$$(1+\rho)\frac{u^{y\prime}(c_t^{y,n})}{u^{o\prime}(c_{t+1}^{o,n})}=1+r_{t+1}(1-\tau_{rt+1}) \tag{9}$$

Also the consumption at the old is

$$c_{t+1}^{o,n}=s_t^n\times(1+r_{t+1}(1-\tau_{rt+1}))+b_t \tag{10}$$

This implies that the following relationship must hold among $c_t^{y,n}$ and $c_{t+1}^{o,n}$ and s_t^n:

$$s_t^n\times(1+\rho)u_y'(c_t^{y,n})-(c_{t+1}^{o,n}-b_t)u_o'(c_{t+1}^{o,n})=0. \tag{11}$$

Before analyzing the effect of accepting immigrants, it would be useful to characterize the initial steady state. Let w^* and r^* be the pre-tax wage rate and the interest rate at the initial steady state, respectively.

Let s^* and N_0^n be the individual saving and the number of old people at the period 0. Let τ_w^* be the wage tax rate at the initial steady state. Let b^* be the social security benefit at the initial steady state.

The initial steady-state economy with zero capital income tax is characterized as follows:

$$s^* = \arg\max_{s_t^n} u^y(w^*(1-t_w^*) - s_t^n) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho}[u^o((1+r^*)s + b^*) + v^o(g^o, g^{ind})] \quad (12)$$

$$w^* = \frac{\partial F(1+\pi_n, s^*)}{\partial L} \ ; \ r^* + \delta = \frac{\partial F(1+\pi_m, s^*)}{\partial K}$$

$$\tau_w^* w^*(1+\pi_n)N_0^n - N_0^n(b^* + g^o) - N_0^n(1+\pi_n)g^y = 0$$

where t_w^*, g^y, g^o,g^{ind},b^* are given.

Also define the steady state level of the consumption and the utility with zero capital income tax as follows:

$$c^{y*} \equiv w^*(1-t_w^*) - s^*;\ c^{o*} \equiv (1+r^*)s^* + b^*;\ u^* \equiv u^y(c^{y*}) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho}[u^o(c^{o*}) + v^o(g^o, g^{ind})]. \quad (13)$$

3.2 Welfare Effect of Accepting Immigrants

In this sub-subsection, I examine whether permanently continuing to accept immigrants will Pareto-improve welfare or not.

Before analyzing the welfare effect of accepting immigrants, first note that the above steady-state with zero capital income tax is efficient since labor supply is inelastic. Now, I consider a case where the government permanently accepts immigrants with a constant ratio. Permanently accepts immigrants with a constant ratio means that $\alpha_0 = 0$ and $\alpha_t = \alpha$ for $t = 1, 2.3, ...$ For the analysis of accepting immigrant permanently, consider the following constrained maximization problem:

Main programming problem (MPP)

$$V(\alpha) = \max_{\{c_t^{y,n}, c_t^{o,n}, s_t^n,\ a_t | t=1,2,..\}} \frac{1}{1+\rho}[u^o(c_1^{o,n}) + v^o(g_o, g^{ind})]$$

$$\text{s.t. } u^y(c_t^{y,n}) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho}[u^o(c_{t+1}^{o,n}) + v^o(g^o, g^{ind})] \geq u^* \quad \text{for } t = 1, 2, ... \tag{14}$$

$$F((1+\pi_n) \times (1+\alpha), s^*) + (1-\delta)s^* \geq$$

$$\{c_1^{y,n} + s_1^n + g^y + g^{ind} + a_1\} \times (1+\pi_n) \times (1+\alpha) + \{c_1^{o,n} + g^o + g^{ind}\} \tag{15}$$

$$F(\{(1+\pi_m) \times \alpha + (1+\pi_n)\}, s_t^n + a_t) + (1-\delta)(s_t^n + a_t) \geq$$

$$\{c_{t+1}^{y,n} + s_{t+1}^n + g^y + g^{ind} + a_{t+1}\}\{(1+\pi_m) \times \alpha + (1+\pi_n)\} + \{c_{t+1}^{o,n} + g^o + g^{ind}\} \quad \text{for } t = 1, 2, ... \tag{16}$$

The above programming problem deserves several comments. First, $V(\alpha)$ is the utility of the cohort 0 at the period 1 when the government accepts immigrant permanently with a constant ratio α. Second, the first constraint is related with Pareto-improvement and it requires that the all cohort except cohort 0 need to have at least as the same utility that they would have at the initial steady state. Second, (15) and (16) are the resource constraints. Instead of the government budget constraint, we use the resource constraint since both are equivalent. Third, the objective function and the constraint are concave with respect to consumption and saving given α. Thus, if the solution of the MPP exist, it is unique. Fourth, in those resource constraints, the tax and social security are not defined. But the once the consumption and saving level are defined, the tax and social security benefit is calculated implicitly. To demonstrate, suppose that c_t^y, c_t^o, s_t and a_t are determined. Then, w_t, r_t, t_{wt}, t_{rt} and b_t are calculated from the following equations:

$$w_t : w_t = \frac{\partial F}{\partial L}((1+\pi_m) \times \alpha + (1+\pi_n)\}, s_t^n + a_t) \tag{17}$$

$$r_t : r_t = \frac{\partial F}{\partial K}((1+\pi_m) \times \alpha + (1+\pi_n)\}, s_t^n + a_t) - \delta \tag{18}$$

$$\tau_{wt} : w_t(1-\tau_{wt}) = c_t^{y,n} + s_t^n \tag{19}$$

$$\tau_{rt} : (1+\rho)\frac{u_y'(c_t^{y,n})}{u_o'(c_{t+1}^{o,n})} = 1 + r_{t+1}(1-\tau_{rt+1}) \tag{20}$$

$$b_t : s_t^n \times (1+\rho)u_y'(c_t^{y,n}) - (c_{t+1}^{o,n} - b_t)u_o'(c_{t+1}^{o,n}) = 0 \tag{21}$$

Let γ_t be the Lagrangian multiplier of (14). Let L and λ_t be the Lagrangian function and Lagrangian multiplier of the resource constraints of (15) and (16), respectively. Let γ_t^* be the Lagrangian multiplier of the utility constraint at period t when $\alpha = 0$. Let λ_t^* the Lagrangian multiplier of the resource constraint of period t when $\alpha = 0$. Then, we have the following observation.

Observation 1

When $\alpha = 0$ the solution of MPP is

$$c_t^y = c^{y*},\ c_0^o = c_t^o = c^{o*}\ ,\ s_t = s^*,\ a_t = 0 \text{ for } t = 1, 2, ... \tag{22}$$

$$\lambda_1^* = \frac{1}{1+\rho} u_o'(c_1^o, g_o, q) \text{ and } \lambda_{t+1}^* = \frac{1+\pi_n}{1+r^*}\lambda_t^* \tag{23}$$

$$\gamma_t^* = \frac{1}{u_y'(c^{y*})}\lambda_t^*(1+\pi_n) \text{ and for } t = 1, 2, ... \tag{24}$$

Proof See appendix A1.

Observation 1 implies that with the constraint $\alpha = 0$, the initial steady state allocation is Pareto-efficient and that it is not possible to have Pareto-improvement from the initial steady state. Now, suppose that the government accepts immigrant permanently with a constant proportion. Whether such acceptance of immigrants Pareto-improves the welfare or not can be analyzed by calculating $V'(\alpha)$ and evaluate at $\alpha = 0$. From the envelope theorem,

$$\begin{aligned} \left.\frac{dV}{d\alpha}\right|_{\alpha=0} &= \left.\frac{\partial L}{\partial \alpha}\right|_{\alpha=0} \\ &= (1+\pi_n)\lambda_1\{\frac{\partial F(1+\pi_n, s^*)}{\partial L} - c_1^{y,n} - s_1^n - g^y - a_t - g^{ind}\} \\ &+ (1+\pi_m)\sum_{t=2}^{\infty}\lambda_t\{\frac{\partial F(1+\pi_n, s_t^n)}{\partial L} - c_t^y - s_t^n - g^y - a_t - g^{ind}\}. \end{aligned} \tag{25}$$

Note that at $\alpha = 0$, $\lambda_t = \lambda_t^*$and $a_t = 0$. From (22), $c_t^{y,n} = c^{y*}$, $s_t^n = s^*$. Thus, we have

$$\left.\frac{dV(\alpha)}{d\alpha}\right|_{\alpha=0} = \left\{(1+\pi_{n)}\lambda_1^* + (1+\pi_m)\sum_{t=2}^{\infty}\lambda_t^*\right\} \times \left\{\frac{\partial F(1+\pi_n, s^*)}{\partial L} - c^{y*} - s^* - g^y - g^{ind}\right\} \tag{26}$$

The first bracket is positive since the Lagrangian multiplier of the resource constraint is positive. As for the second bracket, the first term is the marginal product of labor and it is what the immigrants brings to the economy. It is also the pre-tax wage income of the young at the initial steady state. $c^{y*} + s^*$ is equal to the after tax income of the young individual. $g^y + g^{ind}$ are the private goods that are provided by the government for the young individual Therefore, $c^{y*} + s^* + g^y + g^{ind}$ is the resource allocated to the young while they are young at the initial steady state. This implies that the inside of the bracket is the pre-tax wage minus the resource allocated to the young at the initial steady state. Therefore, the bracket is the amount of an intergenerational transfer from the young to the old. For example, if the inside of the second bracket is equal to zero, it implies that what the young earns is fully allocated to the young(by including pubulically provided private goods).

Proposition1. (MPL condition) *If there is an intergenerational transfer from the young to the old at the initial steady state in the sense that the marginal product of labor of the young is greater than the resource allocated to the young(private consumption, saving plus publicly provided private goods of the young), then accepting immigrants is Pareto-improving for all generations.*

Graphically, Proposition 1 can be explained as follows. To illustrate, consider a simple case where the depreciation rate is 100% ($\delta = 1$) and the fertility rate of the native and immigrant are the same. At period 0, the following resource constraint must hold

$$F((1+\pi_n)N_0^n, N_0 s^*) - N_0^n(c^{o*} + g^o + g^{ind}) - N_0^n(1+\pi_n)(c^{y*} + s^* + g^y + g^{ind}) = 0 \tag{27}$$

By dividing both side by N_0^n, we have

$$F(1+\pi_n, s^*) - (c^{o*} + g^o + g^{ind}) - (1+\pi_n)(c^{y*} + s^* + g^y + g^{ind}) = 0 \tag{28}$$

Note that $F(1+\pi_n, s^*)$ is the GDP per old when the population growth rate is equal to π_n. Now consider the graph of (y, x) where the vertical axis measure the output per old and the horizontal axis measure the one plus the population growth rate. Thus, $x = 1+\pi_n$, $y = F(x, s^*)$. Next, draw the line of (y, x) where y is defined as $y = (c^{y*}+s^*+g^y+g^{ind})x$(see Figure 1). Note that $(c^{y*}+s^*+g^y+g^{ind})x = \frac{(c^{y*}+s^*+g^y+g^{ind})xN_0^n}{N_0}$. $y = (c^{y*} + s^* + g^y + g^{ind})x$ represents the total resource used for the young divided the number of old when $x - 1$ is the population growth rate. This line passes the origin and the slope is $c^{y*} + s^* + g^y + g^{ind}$. At $x = 1 + \pi_n$, the vertical distance of this line represents the total amount of resource used for young divided by the number of old when the population growth rate is equal to π_n. Thus, the difference between $y = F(x, s^*)$ and $y = (c^{y*} + s^* + g^y + g^{ind})x$ at $x = 1 + \pi_n$ represents the amount of resource used for one representative old.

Now suppose that a social planner increases the population growth rate by accepting immigrants permanently. This implies that x moves to right from $x = 1 + \pi_n$. If the slope of $y = F(x, s^*)$ at the $x = 1 + \pi_n$ is greater than $c^{y*} + s^* + g^y + g^{ind}$, then the social planner can maintain the same resource per young (private consumption, saving and governed provided private goods and age independent public goods) and increase the resource per old. Clearly this is the Pareto-improvement. Note that when the government accepts immigrants there is a surplus that is equal to $\partial F(1+\pi_n, s^*)/\partial L - (c^{y*}+s^*+g^y+g^{ind})$ at every period. Also note that $\sum \lambda_t^*$ is the present discounted value of increasing the resource for all periods. Therefore, (26) presents the present discounted value of the surplus of all periods obtained by increasing the population growth rate.

Because the marginal product of labor of the young is the pre-tax wage of the young at the initial steady state and because $c^{y*} + s^*$ is the after-tax income of the young by the definition, $\frac{\partial F(1+\pi_n, s^*)}{\partial L} - c^{y*} - s^*$ is the amount of tax paid by the young. This implies that if the amount of tax paid by the young is greater than the goods provided by the government to the young at the initial steady state, accepting immigrants is Pareto-improving.[5]

Observation 2 *If the tax paid by the young is greater than the publicly provided private goods to the young generation, then accepting immigrant is Pareto-improving.*

Observation 2 implies that the welfare effect of accepting immigrant depends on what the tax new immigrant pays minus what the immigrant receives as publicly provided private goods when they are young. It shows that the social security benefit that the immigrant receives when the are old should not be included for the cost and benefit calculation of accepting immigrant. [6]

[5]Note that the tax condition does not change even in the presence of public goods since an increase of the immigrants does not affect the consumption of public goods by the nature of public goods.

[6]Observation 2 is useful when we need to conduct the cost-benefit analysis of accepting immigrants. When researchers calculate the cost and benefit of accepting immigrants, the inclusion of the social benefits that the immigrant receive is critical to determine the cost of accepting immigrants. Often researchers argues that the social security tax that the immigrants

3.3 Elastic labor Supply, Distorting Taxes and Implementation of Pareto-improvement

The previous subsection has shown that it is Pareto-improving to accept immigrants when there are intergenerational government transfers from the young to the old in the sense that the marginal product of labor of the young is greater than the resource is allocated to the young. With neoclassical production function that exhibits diminishing marginal product, the pre-tax wage decreases and the pre-tax interest rate increases when immigrants come. To Pareto-improve welfare all generations, adjustment of taxes or other policies are needed. However, the previous analysis does not show how taxes or other polices are used while accepting immigrant.

Also, in the previous subsection, I have assumed that the labor supply is inelastic and that there is no capital income tax at the initial steady state. When labor supply is inelastic, capital income tax can be optimal at the initial steady state. But when there are distorting taxes at the initial steady state and when the tax adjustment is conducted for accepting immigrants, it is difficult to distinguish whether Pareto-improvement is achieved through tax changes or the direct effect of accepting immigrant.

In this subsection, I will show that even if labor supply is endogenous and there are distorting taxes at the initial steady state, accepting immigrant Pareto-improve welfare as long as MPL condition is satisfied no matter how wage taxes and capital income taxes are used at the initial steady state. In addition, I shows that with a relatively simple tax adjustment achieve this Pareto improvement while accepting immigrants.

For the analysis, I assume that labor supply is elastic and that the government uses a capital income tax and the wage tax at the initial steady state. We can assume that those taxes are not the second best optimal. Let l^* be the labor supply of a young native or immigrant when he/she faces the same wage rate

pays and the social security benefit that immigrants receives should included for the cost-benefit calculation(traditional view). However, Sinn(2001) and Razin(2004) showed that the social security benefit that the immigrant receive should not be included. If the social security benefit that immigrant receives is included, then the social security tax that children of the immigrant pays should be included since the social security benefits of the old is paid by the children of the immigrants through the social security tax. But if the social security is roughly a pay-as-you-go system, then the amount of the social security benefit of the immigrants receives and the amount of the social security tax that the children of the immigrants pay is roughly balanced. This implies that the social security benefit should not be included and only the social security tax that the immigrants pays should be included. Observation 2 supports the logic of Sinn and Razin.

and the tax rate as at the initial steady state. Note that since we assume that the preference of immigrant and native are the same and that immigrant and native have the same efficiency unit of labor, we do not put a superscript j on l^*.

To achieve Pareto-improvement by accepting immigrants, we assume that the government sets the wage tax rate and the interest tax rate at period t in the following way:

$$w_t(1-\tau_{wt}) = w^*(1-\tau_w^*) \text{ and } r_t(1-\tau_{rt}) = r^*(1-\tau_r^*) \tag{29}$$

In other words, the government sets the tax rates so that after tax wage and interest rate after the acceptance of immigrants are equal to after tax wage rate and interest rate at the initial steady state. Also assume that the government gives the same social security benefit as at the initial steady state. This implies that $b_t = b$ for $t = 1, 2, 3...$ When the government sets taxes and social security benefit in this way, saving behavior and labor supply behavior do not change because the consumers have the same after tax wage and interest rate as at the initial steady state. If the government provides at least as the same level of publicly provided private goods, the levels of the utility of all cohorts are at least as the same as at the initial steady state.

As for the size of immigration, motivated by Proposition 1, assume that MPL condition is satisfied at the initial steady state:

$$l^* \frac{\partial F(l^*(1+\pi_n), s^{n*})}{\partial L} > c^{y*} + s^* + g^y + g^{ind} \quad . \tag{30}$$

The LHS of the above equation is a marginal increase of the output due to one unit increase of population. In the RHS is $c^{y*} + s^*$ is the after-tax income of the of each young individual and $g^y + g^{ind}$ is the resource used by the government for the young individual. Thus RHS is the amount of resource allocated to the young. The result in the previous section shows that, starting from zero immigrants, a marginal increase of immigrants Pareto-improve welfare if MPL condition is satisfied. But that result does not implies unlimited acceptance of immigrants always Pareto-improve welfare. I assume the government accept immigrant and choose α so that MPL condition is not violated. This implies that

$$l^* \frac{\partial F(l^* \times (1+\alpha) \times \{(1+\pi_n)N_{t-1}^n + (1+\pi_m) \times N_{t-1}^m\}, (N_{t-1}^n + N_{t-1}^m)s^*)}{\partial L} \geq c^{y*} + s^* + g^y + g^{ind} \quad (31)$$

Note that $(1+\pi_m) \times N_{t-1}^m + (1+\pi_n)N_{t-1}^n$ is the population of young native at the period t. With the immigration policy α, $\{(1+\alpha) \times \{(1+\pi_n)N_{t-1}^n + (1+\pi_m) \times N_{t-1}^m\}$ is the total population of the young native and immigrant. Using the homogeniety of production function and $N_{t-1}^m = \alpha N_{t-1}^n$, (31) can be written as

$$l^* \frac{\partial F(l^* \times \{(1+\pi_n) + (1+\pi_m) \times \alpha\}, s^*)}{\partial L} \geq c^{y*} + s^* + g^y + g^{ind} \quad (32)$$

When the government accepts immigrant and adjusts the wage tax rate and capital income tax rate so that after tax rate and after tax interest rate are the same as at the initials steady state, the government can accumulate the government saving as I prove below. This implies that it is possible that at some point, the marginal product of capital (MPK) becomes equal to the golden level of capital stock per capita. But when MPK becomes equal to the golden rule level, it is clearly better to use the all government surplus to increase the supply of publicly provided private goods rather than to increase the government saving balance. Thus, I assume that as long as MPK is higher than the golden rule level, the government uses some of the government budget surplus to increase the government saving balance and uses the rest to increase the supply of publicly provided private goods. When MPK reaches the golden rule level, the government uses all surplus to increase the publicly provided private goods. Thus, we have the following MPK condition:

$$\frac{\partial F}{\partial K} \geq \delta + (1+\pi_m)\alpha + \pi_n \quad (33)$$

Now let $\partial F/\partial L$ be w_t and let τ_{wt} be the wage tax rate at period 1 when the immigrants are accepted. Let τ_{rt} be the tax rate on the interest income at period t. Let τ_r^* be the capital income tax rate at the initial steady state. The question that we examine is whether such taxes are feasible from the point of the government budget constraint. To check the feasibility of such taxes, consider the net government budget surplus at the period 1, SP_1.

$$SP_1 = w_1\tau_{w1}(l^*N_1^n + l^*N_1^m) + r_1\tau_{r1}s^*N_0^n - N_0^n \times (b + g^o + g^{ind}) - (N_1^n + N_1^m) \times (g^y + g^{ind}) \tag{34}$$

Note that from (29)

$$\tau_{w1} = 1 - \frac{w^*(1-\tau_w^*)}{w_1}\ ,\ \tau_{r1} = 1 - \frac{r^*(1-\tau_r^*)}{r_1} \tag{35}$$

By substituting τ_{w1} and τ_{r1} into SP_1 and using homogeneity of production function, we have (see appendix B1)

$$SP_1 = \frac{N_0^n}{l^*}\int_{l^*(1+\pi_n)}^{l^*(1+\pi_n)(1+\alpha)} [l^*\frac{\partial F(z,s^*)}{\partial L} - (c^{y*} + s^*) - (g^y + g^{ind})]dz \tag{36}$$

Note that from the MPL conditions (30) and (31), the inside of the integration is positive for $z \in [l^*(1+\pi_n), l^*(1+\pi_n)(1+\alpha)]$. This means that this tax plan is feasible in the period 1. The government can use some of the surplus of the budget to increase the supply of publicly provided private goods and accumulate the rest as the government saving. Let $a_1 > 0$ be the balance of the government savings per young population at the end of period 1. How about the net budget surplus in period 2, SP_2 ?

Note that the consumers who are born at the period 1 will save as the consumers at the initial steady state economy since the consumers born in the period 1 face the same after-tax wage and interest rate as the consumers at the initial steady state under the proposed tax policy. This implies that $s_1 = s^*$ for both consumers born at the period 1 and immigrants who come at the period 1. The total capital stock at the period 2 are $\sum_j N_1^j(s^* + a_1)$. SP_2 becomes

$$SP_2 = (N_2^n + N_2^m) \times (w_2\tau_{\tau w2} - g^y - g^{ind}) + \sum_j N_1^j \times (r_2\tau_{r2}s^* - b - g^o - g^{ind}) + (1+r_2)a_1\sum_j N_1^j. \tag{37}$$

Note that the pre-tax wage at period 2, w_2, and the pre-tax interest rate at period 2, r_2, are equal to

$$w_2 \equiv \frac{\partial F(l^*(N_2^n + N_2^m), (N_1^n + N_1^m)(s^* + a_1))}{\partial L} \text{ and } r_2 \equiv \frac{\partial F(l^*(N_2^n + N_2^m), (N_1^n + N_1^m)(s^* + a_1))}{\partial K} - \delta \tag{38}$$

Again the government sets τ_{w2} and τ_{r2} so that after tax wage and after tax interest rate becomes the same as the ones at the initial steady state. This implies $\tau_{w2} = 1 - \frac{w^*(1-\tau_w^*)}{w_2}$, $\tau_{r2} = 1 - \frac{r^*(1-\tau_r^*)}{r_2}$. Thus, SP_2

becomes (See appendix B2)

$$SP_2 = \int_{(N_1^n+N_1^m)s^*}^{(N_1^n+N_1^m)(s^*+a_1)} \{\frac{\partial F(l^* N_2^n + l^* N_2^m, z)}{\partial K} + (1-\delta)\}dz$$
$$+ (1+\alpha)\frac{N_1^n}{l^*} \int_{l^*(1+\pi_n)}^{l^*\{(1+\pi_n)+(1+\pi_m)\alpha\}} \{l^* \frac{\partial F(z, s^*)}{\partial L} - w^* l^*(1-\tau_w^*) - (g^y + g^{ind})\}dz \tag{39}$$

The first term of (39) measures the welfare gain that arises from the additional saving that the government accumulated at the end of period 1. The second term measures the welfare gain that arises from the increased population growth rate in the presence of PYGO social security system. From MPK condition (33), the inside of the first integration is positive. From (32), the inside of the second integration is positive. Thus, SP_2 is positive and the government can implement the proposed tax policy. Again, at the end of period 2, the government can use some of the above surplus to increase the supply of publicly provided private goods and use the rest to increase the balance of the government savings. Similarly, the government surplus at the period t becomes

$$SP_t = \int_{(N_{t-1}^n+N_{t-1}^m)s^*}^{(N_{t-1}^n+N_{t-1}^m)(s^*+a_{t-1})} \{\frac{\partial F(l^* N_t^n + l^* N_t^m, z)}{\partial K} + (1-\delta)\}dz$$
$$+ \frac{(1+\alpha)N_{t-1}^n}{l^*} \int_{l^*(1+\pi_n)}^{l^*\{(1+\pi_n)+(1+\pi_m)\alpha\}} \{l^* \frac{\partial F(z, s^*)}{\partial L} - c^{y*} - s^* - (g^y + g^{ind})\}dz \tag{40}$$

where a_t is the balance of government saving per young population at the end of period t-1. Again the government uses some of the surplus for increasing publicly supplied private goods and the rest for the government saving. This implies that $SP_t > 0$ for all $t = 1, 2, ...$ Thus, we have the following Proposition 2.

Proposition 2. *Consider an economy where the labor supply is elastic and that the (optimal or non-optimal) wage and interest taxes are used at the initial steady state. If MPL condition is satisfied at the initial steady state, accepting immigrants with tax rule (29) Pareto-improves welfare for all generations.*

3.4 Government Saving and the Golden Rule

This subsection examines the capital stock path and government saving path when immigrants are accepted. To examine the government saving path, we need to specify how much of the government surplus,

SP_t, is used for the government saving . To simplify the argument, assume that the surplus that arises from the increased government saving in period $t-1$, which is first integration of SP_t is used for the government saving at the period t. Note that the government could use some part of the second integration in SP_t, the surplus generated directly from the increased immigration. Thus, my assumption is a conservative value of the government saving. Even with this conservative level of the government saving, I will show that the economy reaches the gold rule level of capital stock per capita within finite time. The government saving per young population at the end of period t for $t \geq 2$, a_t, becomes

$$a_t = \frac{1}{N_t^n + N_t^m} \int_{(N_{t-1}^n + N_{t-1}^m)s^*}^{(N_{t-1}^n + N_{t-1}^m)(s^* + a_{t-1})} \{\frac{\partial F(l^* N_t^n + l^* N_t^m, z)}{\partial K} + (1-\delta)\} dz \tag{41}$$

Note that from (41) and $N_t^m = \alpha N_t^n$, $a_t \geq a_{t-1}$. Thus, a_t is increasing over time as long as a_t is determined according to equation (41). By rewriting the above equation, we have

$$\begin{aligned}
a_t &= \frac{1}{N_t^n(1+\alpha)} \int_{(N_{t-1}^n + N_{t-1}^m)s^*}^{(N_{t-1}^n + N_{t-1}^m)(s^* + a_{t-1})} \{\frac{\partial F(l^* N_t^n(1+\alpha), z)}{\partial K} + (1-\delta)\} dz \\
&= \frac{1}{N_t^n(1+\alpha)} [F(l^* N_t^n(1+\alpha), (1+\alpha)N_{t-1}^n(s^* + a_{t-1})) - F(l^* N_t^n(1+\alpha), (1+\alpha)N_{t-1}^n s^*) + (1-\delta)(1+\alpha)N_{t-1}^n a_{t-1}] \\
&= \frac{1}{1+\pi_n + \alpha(1+\pi_m)} \{F(l^*\{1+\pi_n + \alpha(1+\pi_m)\}, s^* + a_{t-1}) - F(l^*\{1+\pi_n + \alpha(1+\pi_m)\}, s^*) + (1-\delta)a_{t-1}\} \\
&\equiv Q(a_{t-1})
\end{aligned}$$

Figure 2 shows that the graph of $a_t = Q(a_{t-1})$. $Q(a_t)$ is zero at $a_{t-1} = 0$ and $Q(a_t)$ is concave due to the diminishing marginal product of capital. The slope of $Q(a_t)$ at $a_t = 0$ is

$$Q'(0) = \frac{1}{(1+\pi_m) \times \alpha_t + (1+\pi_n)} (\frac{\partial F(l^*\{(1+\pi_m) \times \alpha + (1+\pi_n)\}, s^*)}{\partial K} + 1 - \delta) \tag{42}$$

Because of the assumption (33), $Q'(a_t)$ at $a_t = 0$ is greater than one. Thus, the $a_t = Q(a_{t-1})$ and 45 degree line intersect at $a_t = 0$ and a^* where $a^* > 0$. Let a^{**} be the point where $Q'(a^{**}) = 1$. This implies that at a^{**}

$$\frac{\partial F(l^*\{(1+\pi_m) \times \alpha + (1+\pi_n)\}, s^* + a^{**})}{\partial K} + 1 - \delta = (1+\pi_m) \times \alpha + (1+\pi_n).$$

In other words, at a^{**} the golden rule is satisfied. Note that the government can choose a_1 since the surplus at the period 1 is strictly positive. From the graph of $a_t = Q(a_{t-1})$, a_t keeps increasing starting from a small $a_1 > 0$. Before it reaches a^*, it reaches a^{**} within a finite time. This implies that the economy reaches the golden rule level of capital stock per capita within a finite time.

Proposition 3. (Reaching Golden rule) *Suppose that the PYGO social security system is initially used and that MPL condition is satisfied. Then, by accepting immigrants and using the proposed tax and government saving policy, the government can make the economy reach the golden rule level within in a finite time in a Pareto-improving way.*

Note that in the above analysis, I assume that only the first integration of SP_t the surplus that arises from the increased capital stock at the period $t-1$, is used for the government saving at the period t. However, the second integration of SP_t, the surplus which directly arises from accepted immigrants, also can be used for the government saving Thus, the government can shorten the time to reach the golden rule by using the surplus that directly arises from the accepted immigrants.

3.5 Intra-redistributional Channel and Difference of Productivities and Preferences

When immigrant earns less than the native but immigrants receive the same amount of publicly provided private good as a native, accepting immigrant could decrease the welfare of the native since it means that more resources is taken from the native and used for immigrant. This is intra-redistributional channel regarding the effect of accepting immigrant. A similar redistribution channel could also happen when the behavior of immigrant and native are different and the labor supply or saving of immigrants are those of natives. To analyze this redistributional channel of accepting immigrant, consider again a permanent immigrants policy where the size of accepted immigrants is $100 \times \alpha$ percent of the native. Let ($c^{y,j*}$,$c^{o,j*}$, s^{j*}, l^{j*}), be the consumption at the young, the consumption at the old period, saving and labor supply of native($j = n$) or the immigrant($j = m$) when the wage rate and the interest rate are at the initial steady state level and the tax rate on interest and wage are also at the initial steady state level. Since I assume that the preferences and the productivities of the native and immigrant are different in this section, I put superscript j even for the variables at the steady state situation. For conditions that guarantees accepting immigrant Pareto-improve welfare, first I assume that two MPL conditions, (30) and (31), which are used in the previous subsection, are satisfied.

Next we need an index that parameterizes intra-redistribution between the native and immigrant. At the period t, we calculate the difference of the contribution to government budget between the real immigrant and hypothetical immigrant who are identical with native. More specifically,

$$\phi w_1 \tau_{w1} l^{m*} N_t^m + r_t \tau_{r1} s^{m*} N_{t-1}^m - N_{t-1}^m \times (b + g^o + g^{ind}) - N_t^m \times (g^y + g^{ind}) \quad (43)$$

is the real contribution to the government budget by the real immigrant when the immigrant faces the same after tax wage rate and interest rate as at the initial steady state. Next, consider

$$w_1 \tau_{w1} l^{n*} N_t^m + r_t \tau_{r1} s^{n*} N_{t-1}^m - N_{t-1}^m \times (b + g^o + g^{ind}) - N_t^m \times (g^y + g^{ind}) \quad (44)$$

which is the hypothetical contribution to the government budget by the hypothetical immigrants who are identical to the native in productivity and preferences. Note that in (44), the number of young immigrant is still N_t^m and the number of old immigrants is N_{t-1}^m. It is called hypothetical contribution because it assumes that immigrant behave in the same way as the native regarding labor supply and saving and that the productivity of immigrant is the same as that of the native ($\phi = 1$). Given those two contributions to the government budget, define Redistribution index$_t$ as the difference of those two contributions to the government budget, (43) and (44):

$$\begin{aligned}\text{Redistribution index}_t = &\{\phi w_1 \tau_{w1} l^{m*} N_t^m + r_t \tau_{r1} s^{m*} N_{t-1}^m - N_{t-1}^m \times (b + g^o + g^{ind}) - N_t^m \times (g^y + g^{ind})\} \\ &- \{w_1 \tau_{w1} l^{n*} N_t^m + r_t \tau_{r1} s^{n*} N_{t-1}^m - N_{t-1}^m \times (b + g^o + g^{ind}) - N_t^m \times (g^y + g^{ind})\}\end{aligned}$$

Note that, by construction, Redistribution index$_t$ is equal to zero if immigrant's preference and productivity are the same as these of the natives.

Now given this Redistribution index$_t$, we can define the modified suprplus, $MdSP_t$ as follows:

$$MdSP_t = SP_t + \text{Redistribution index}_t \quad \text{where } l^* = l^{n*} \text{ in } SP_t$$

Intutively, the modified suruplus is the sum of the redistribution from the native to immigrant and the government budget surplus that the government receives when the government accepts hypothetical immigrants who are identical with natives.

Proposition 4 *Consider a immigratio policy where the government accept* $\alpha \times 100$ *percent of native permanently. Assume that accepted immigrants earn less than the native and the preference of labor supply and saving of the immigrants are different from the native. If* $MdSP_t > 0$ *for all* t, *it is possible to Pareto-improve welfare by accepting immigrants. .*

Proof (See Appendix B3)

Proposition 5 *If the conditions of proposition 4 are satisfied, then the government can make the economy to reach the golden rule within a finite time.*

Proof (See Appendix B4)

Note that $MdSP_t > 0$ is likely to be satisifed if immigrant and native are identical or the size of intern-generational transfer by PYGO social security is larger than the intro-redistribution between immigrants and the native[7].

4 Quantifying the Welfare Gain of Accepting Immigrants in the Presence of PYGO Social Security

Propositions 1 and Propostion 2 in the previous section show that accepting immigrants can Pareto-improve the welfare of all generations if there are inter-generational transfers in the sense that the marginal product of labor of the young is greater than the sum of consumption, including publicly provided private good and saving of the young (MPL condition). Furthermore, Proposition 3 shows that if the government can save some of the welfare gain as the government saving, it is possible to make the economy reach the golden rule level of capital stock in a Pareto-improving way within a finite time. This is a sharp contrast to the previous literature of the social security reform that shows that to increase the capital stock of

[7] To determine whether the intergenerational transfer is larger than the intra-redistrution, we need a more realistic large scale model. This is one of motivations of the analysis in the next section.

the economy in the presence of PYGO social security system, some generation must bear the double burden(Geanakoplos, John, Olivia S. Mitchell and Stephen P. Zeldes 1998). In addition, the proposition 4 and 5 show that even if immigrant earn less than the native, accepting immigrants Pareto-improve welfare when the intra-redistribution is not so large as the inter-generational transfer,

There are several questions to the propositions 1-5, however. First, those propositions are based on two-period overlapping generation model. In a realistic multi-period overlapping generation model, it might not be possible for an economy to reach the golden rule level of capital stock per capita in a finite time in a Pareto-improving way by accepting immigrants. Second, although the proposition 3 shows that the economy reaches the golden rule level of capital stock per capita in a finite time, practically it might take a quite long time, for example 1000 years, to reach the golden rule although the economy still reaches within a finite time. Third, the propositions 1-5 are silent about the quantitative welfare effect. Given that it might take a quite long time to reach the golden rule, the welfare gain of accepting immigrants can be very small. In addition, proposition 4 and 5 is silent about what degree of wage difference between immigrant and native is allowed to achieve Pareto-improve welfare when immigrants are accepted.

This section answers those questions. To answer those questions, I use the computational overlapping generation model developed by Auerbach and Kotlikoff(1987). I assume that the model economy consists of overlapping generations where each generation lives for 80 periods with increasing probability of death. I assume that the model economy is similar to the US economy in several dimension[8]. In the analysis, I assume that initially the model economy is on the balanced growth path and the immigrant/native ratio (INR) is similar to the INR that the US 2000 census data indicates. Then, I will examine whether it is possible to Pareto-improve the welfare of all generations in the model economy by increasing the INR by a reasonable size. In addition, I examine how long it takes for the model economy to reach the golden-rule level in a Pareto-improving way and quantify the Pareto-improving welfare gain. To check the robustness, I recalculate the by changing the value of the following parameters the target immigrant-native ratio, the

[8] However, the model economy is different in several important dimensions as well. For example, the model economy does not include the open economy side such as international trade and capital mobility. The model economy does not incorporate the human capital accumulation of the natives and immigrants.

share of the surplus for the government saving, the replacement, the coefficient of the constant relative risk aversion, the discount factor and the initial population growth rate and immigrant's earning level.

4.1 The model economy: Auerbach and Kotlikoff model with immigration

Agents will show up in the model from age 1. I assume that age 1 corresponds to age 20 in real life. From age 1 until age 45 they work. At 46, they retire. At each age, they die with some probability and they can live unitl 80 years old at maximum. For $i \geq 2$, let p_i be the probability that an agent is alive at the age i given that he or she is alive until age $i-1$. Because the lack of data, I assume that the p_i is the same for the native and immigrant. To simplify the notation, I also assume that $p_1 = 1$.[9] As in the previous section, let j be the index indicating an agent is native ($j = n$) or immigrant ($j = m$). An agent who enters the model at the period t maximizes the following utility function:

$$\max \sum_{i=1}^{45} \beta^i \prod_{q=1}^{i} p_q \left\{ \frac{[(c_{t-1+i}^{i,j})^{\alpha}(1-l_{t-1+i}^{i,j})^{1-\alpha}]^{1-\gamma}}{1-\gamma} + g_{t-1+i}^{i} \right\} + \sum_{i=46}^{80} \beta^i \prod_{q=1}^{i} p_q \left\{ \frac{[c_{t-1+i}^{i,j}]^{\alpha(1-\gamma)}}{1-\gamma} + g_{t+1-i}^{i} \right\} \tag{45}$$

where $c_{t-1+i}^{i,j}$ and $l_{t+1-i}^{i,j}$ are the amount of the private consumption and the labor supply of type j agent at the period $t-1+i$. g_{t+1-i}^{i} is the publicly provided private good to the age i agent in the period $t+1-i$. I assume that the amount of g_{t+1-i}^{i} is chosen by the government. The government does not discriminate immigrants regarding the supply of the publicly provided private goods. The budget constraint of an agent at age i is

$$s_{t-2+i}^{i-1,j}(1 + r_{t+1-i}(1-\tau_{r,t+i})) + (1-\tau_{w,t+1-i})w_{t-1+i}^{e} H^{i,j} \times l_{t-1+i}^{i,j} = c_{t-1+i}^{i,j} + s_{t+1-i}^{i,j} \text{ for } 1 \leq i \leq 45$$

$$s_{t-2+i,}^{i-1,j}(1 + r_{t+1-i}(1-\tau_{rt-1+i})) + b_{t-1+i}^{i,j} = c_{t+1-i}^{i,j} + s_{t-1+i}^{i,j} \text{ for } 46 \leq i \leq 80$$

$$s_{t-1+i}^{i,j} = 0 \text{ for } i = 0 \text{ and } s_{t+1-i}^{i,j} \geq 0 \text{ for } 1 \leq i \leq 80$$

where $H^{i,j}$ is the efficient unit of human capital of type $j$at age i and $H^{i,j} > 0$ for $1 \leq i \leq 45$ and $H^{i,j} = 0$ for $i \geq 46$. w_t^e is the wage rate for one efficient unit of labor at the period t. I assume that an individual cannot have a negative saving balance. Once an individual dies, the government imposes 100 percent

[9] Infant and child mortality is defined in (46).

bequest tax. $b^{i,j}_{t-1+i}$ is the social security benefit for type j of age i that is given at the period $t-1+i$. For $i \leq 45$, $b^{i,j}_{t-1+i} = 0$ and for $i \geq 46$, $b^{i,j}_{t-1+i}$ is determined as follows:

$$b^{i,j}_{t-1+i} = 12 \times RR \times AIME^j(t) \text{ and } AIME^j(t) = \frac{\sum_{i=1}^{45} w^e_{t-1+i} l^{i,j}_{t-1+i} H^{i,j}}{45 \times 12}$$

where RR is the replacement and $AIME^j(t)$ is the average income monthly index of the cohort who become age 1 at the time t.

For the production function, I assume that the economy's aggregate production can be described as Cobb-Douglas production:

$$Y_t = K_t^{\theta}(E_t L_t)^{1-\theta} \text{ and } \mu = (E_{t+1} - E_t)/E_t$$

where θ is the capital share and E_t is a parameter to represent a technology level. μ is income per capita growth rate. L_t is the efficient unit of labor supply at the period t. L_t is defined as follows:

$$L_t = \sum_{i=1}^{45} \sum_{j=n,m} H^{i,j}_i N^{i,j}_t \text{ and } N^{i,j}_t = N^{i-1,j}_{t-1} p_i$$

where $N^{i,j}_t$ is the population of age i of type j at the period t.

As for immigrants, I assume that all immigrants will come to the host country at age 1. $N^{1,n}_t$ is determined by the fertility rate of immigrant and native. Let $\eta^{i,j}_j$ be the age specific fertility of type j at age i. A person can not have a child physically from a certain age, and it is assumed that $\eta^{i,j}_j = 0$ for those ages. Let d be the sum of infant and child mortality. The number of children born at time t is $\sum_{i=1}^{80} N^{i,m}_{t,i} \eta^{i,m}_m + \sum_{i=1}^{80} N^{i,n}_t \eta^{i,n}$. After 20 years, they will show up in the model as agents with age 1. Thus, we have

$$N^{1,n}_{t+20} = (1-d) \times \left\{ \sum_{i=1}^{80} \eta^{i,m} N^{i,m}_t + \sum_{i=1}^{80} \eta^{i,n} N^{i,n}_t \right\} \tag{46}$$

Since $\Pi^i_{q=1} p_q$ is the probability that an individual is still alive at age i given that an agent is alive at age 1, $N^{i,j}_t = N^{1,,j}_{t-(i-1)} \times \Pi^i_{q=1} p_q$. Then, the equation on $N_{t+20,1}$ becomes

$$\frac{N^{1,n}_{t+20}}{1-d} = \sum_{i=1}^{80} \eta^{i,m} N^{1,m}_{t-(i-1)} \times \Pi^i_{q=1} p_q + \sum_{i=1}^{80} N^{1,n}_{t-(i-1)} \eta^{i,n} \times \Pi^i_{q=1} p_q \tag{47}$$

Now consider the steady state of the population where the ratio between $N^m_{t,1}$ and $N^n_{t,1}$ is constant. This implies that $N^m_{t,1}$ and $N^n_{t,1}$ grow at the same rate σ. Therefore, at the initial steady state, we have $N^{1,n}_{t+20} = N^{1,n}_t(1+\sigma)^{20}$ and $N^{1,j}_{t-(i-1)} = N^{1,j}_t/(1+\sigma)^{i-1}$ for $j = i, m$. Thus, (26) becomes

$$\frac{N^{1,n}_t(1+\sigma)^{20}}{1-d} = \sum_{i=1}^{80} \frac{N^{1,m}_t}{(1+\sigma)^{i-1}}\eta^{i,m} \times \Pi^i_{q=1}p_q + \sum_{i=1}^{80} \frac{N^{1,n}_t}{(1+\sigma)^{i-1}}\eta^{i,n} \times \Pi^i_{q=1}p_q$$

Let the steady state value of $N^{1,m}_t/N^{1,n}_t$ be f. Solving for f, we have

$$\frac{(1+\sigma)^{20}}{1-d} - \sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i-1}}\eta^{i,n} \times \Pi^i_{q=1}p_q = f\sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i-1}}\eta^{i,m} \times \Pi^i_{q=1}p_q$$

$$f = \frac{\frac{1}{1-d} - \sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i+19}}\eta^{i,n} \times \Pi^i_{q=1}p_q}{\sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i+19}}\eta^{i,m} \times \Pi^i_{q=1}p_q} \tag{48}$$

(48) says that the steady state immigrant-native ratio is determined once the annual population growth rate and the fertility rates of native and immigrant are set. Conversely, given the fertility rates of immigrant and native, we can choose σ so that the resulting immigrant/native ratio is consistent with the data.

The capital stock at the period t is the sum of the balance of individual savings and government saving. Let a_{t-1} be the balance of the government asset (or debt if it is negative) per capita at the end of period $t-1$. Then, the total capital stock at the period t is

$$K_t = \sum_{i=1}^{80}\sum_{j=n,m} N^j_{t-1,i}s^j_{t-1,i} + a_{t-1}\sum_{i=1}^{80}(N^n_{t-1,i} + N^m_{t-1,i})$$

The (efficient unit)wage rate at time t, w_t and the pre-tax interest rate at time t, r_t, are determined as

$$w_t = (1-\theta)K^\theta_t E^{1-\theta}_t L^{-\theta}_t \qquad \text{and} \qquad r_t = \theta K^{\theta-1}_t E^{1-\theta}_t L^{1-\theta}_t$$

The government budget constraint at the initial balance growth path, Then the government budget constraint on the initial balanced growth pat is

$$\tau_{wt}w_tL_t + \tau_{rt}r_t\sum_{i=2}^{80} p_iN^{i-1,n}_{t-1,i}s^{i-1,n}_{t-1} + (1+r_t)\sum_{i=2}^{80}(1-p_i)N^{i-1,n}_{t-1,i}s^{i-1,n}_{t-1}$$
$$-\sum_{i=1}^{80} N^{i,n}_{t,i}g^*_i(1+\mu)^t - \sum_{i=46}^{80} b^{i,n}_t \times N^{i,n}_t = 0 \tag{49}$$

When the government accepts immigrants, the wage rate decreases and the interest rate increases. To Pareto-improve welfare, I assume that the government keeps the after tax wage rate and interest rate the same as at the initial balance growth path. More specifically, let τ_{wt} and τ_{kt} be the wage tax rate and the interest tax rate after accepting immigration, respectively. Then, the wage tax rate τ_w and interest tax rate τ_k are set as follows:

$$w_t(1-\tau_{wt}) = w_t^*(1-\tau_w^*) \text{ and } r_t(1-\tau_{rt}) = r^*(1-\tau_r^*) \tag{50}$$

When the taxes are set according to equation (50) , there is a surplus for the government budget even if the government spends the same amount of publicly provided private good per person as at the initial balance growth path. The government can use this surplus for increasing the government saving or increasing the level of publicly provided good. Let V be the distributional parameter that indicates how much percentage of the budget surplus is used for the government saving. Then, the balance of the government saving at the period t is

$$a_t \sum_{i=1}^{80}(N_t^{i,n} + N_t^{i,m}) = a_{t-1} \sum_{i=1}^{80}(N_t^{i,n} + N_t^{i,m}) + V \times SP_t \tag{51}$$

$$\begin{aligned} SP_t = \tau_{wt} w_t L_t + \tau_{rt} r_t \sum_{i=1}^{80} \sum_{j=n,m} p_i N_{t-1}^{i-1,j} s_{t-1}^{i-1,j} + (1+r_t) \sum_{i=1}^{80} \sum_{j=n,m} (1-p_i) N_{t-1}^{i-1,j} s_{t-1}^{i-1,j} \\ - \sum_{i=1}^{80}(N_t^{i,n} + N_t^{i,m}) \times g_i^*(1+\mu)^t - \sum_{i=46}^{80} \sum_{j=n,m} b_t^{i,j} \times N_t^{i,j} \end{aligned} \tag{52}$$

The rest of the government surplus, $(1-V) \times SP_t$ is used to increase the amount of publicly provided private good. Let $\widetilde{g}_t^i$ be the increased publicly provided private good for age i at the period t. Then, the sum of the increased amount of publicly provided private good must be equal to $(1-V) \times SP_t$.

$$\sum_{i=1}^{80}(N_t^{i,n} + N_t^{i,m}) \widetilde{g}_t^i = (1-V) \times SP_t \tag{53}$$

I assume that the ratio of the increased amount of the publicly provided private good are same for all ages:

$$\frac{\widetilde{g}_t^i}{g_i^*(1+\mu)^t} = \text{constant for all } i \tag{54}$$

where $g^{i*}(1+\mu)^t$ is the amount of publicly provided private good at the initial balanced growth path. Then, the amount of publicly provided private goods after the acceptance of immigrants,g_t^i, becomes

$$g_t^i = g^{i*}(1+\mu)^t + \widetilde{g}_t^i. \tag{55}$$

4.2 Thought Experiments and Parameters Values for Simulation

For the age-nativity specific fertility, I use the CPS 2000 June supplement, which is also used by the census bureau for the calculation on the age-nativity specific fertility. Figure 3 shows the average of the total number of births by each woman's age. Figure 3 shows that immigrant women have more total number of births than native women at all ages. From this figure, I calculate $\eta^{i,n}$ and $\eta^{i,m}$, the age specific birth rate for native and immigrant women. This is shown in the Table 1. For d, the sum of infant and child mortality, I set 1.7 percent from Vital Statistics of US 1993.

For the age specific government expenditure, g^i, I follow Storesletten (1995) and Auerbach, Kotlikoff Hagememann and Nicoletti (1989). For $1 \leq i \leq 10$, $g^i = g^y$ and for $11 \leq i \leq 45$, $g^i = g_m$. For $46 \leq i \leq 80$, $g^i = g_o$. I assume that g^y, g^m and g^o are 24.5%,13.4% and 23.2 % of GDP per capita at the initial balanced growth path.

For the coefficient the relative risk aversion, γ, the literature has not found the precise estimate. Auerbach and Kotlikoff(1986) and Storesletten (2000) assumed that γ is 4. Nishiyama and Smetter (2007) set γ equal to 2. I assume that $\gamma = 3$ and check the robustness with $\gamma = 2$ and $\gamma = 4$. For time preference rate, β, following Hurd(1989) and Storesletten(2000), I assume that $\beta = 1.011$. Higher β implies that higher saving and higher capital/output ratio. To check the sensitivity of my results, I also calculate with $\beta = 0.98$ and $\beta = 0.96$. For the leisure share in the utility function, α, I assume that it is 0.33.

As for the production side, I assume that the depreciation rate, δ, is equal to 0.047 according to Storesletten. For the capital share in the production function, θ, I set $\theta = 0.4$. For technological progress, I assume that income per capita growth rate, μ, is 0.015 following Storesletten.

For the human capital profile of the native, H_i^s, I take the value from Auerbach and Kotlikoff.

$$H^{i,n} = \exp(4.47 + 0.033 \times i - 0.00067 \times i^2) \quad \text{for } 1 \le i \le 45 \tag{56}$$

$$H^{i,n} = 0 \quad \text{for } 46 \le i \tag{57}$$

For the human capital profile of the immigrants, Storesletten(1995) showed that immigrants earn 15.7 percent lower than the native on average[10]. Similary, using CPS June 2000 supplement, my calculation shows that immigrants earn 10 percent lower than the native. Based on those estimates, initially I assume that efficient units of human capital of immigrant is 84.3 percent of that of the native and that $H^{i,m} = 0.843 \times H_i^n$ in the benchmark calculation. To examine the robustness of my results, I change human capital level from 84.3 % to 89.3 percent or 79.3 percent and re-check the results. For the mortality profile, p_i, I take the values from Nishiyama and Smetter (2007).

For the capital income tax, I take the value from Nishiyama and Smetter (2007) and it is assumed that $\tau_k = 0.28$. For the social security benefit level, higher replacement means that higher intergenerational income redistribution, which leads to higher welfare gain of increasing the size of immigrant. Following Auerbach and Kotlikoff, in the benchmark case I set the replacement is equal to 0.6 and check the robustness of my results by varying it from 0.6 to 0.55 and 0.5.

As for the initial government level of debt or asset, different authors set different levels. Storesletten considered only the government debt and assumed that the initial debt level is 50 percent of the initial GDP. With his estimate of the initial capital-output ratio being 2.4, it implies that the government debt is about 20 percent of the private capital. Nishiyama and Smetter considered not only the government debt but also the government asset by using the BEA information on the government fixed capital. They assume that at the initial steady state, the government has a positive asset in net and assume that it is 10 percent of the total private capital. This naturally leads to a higher capital output ratio at the initial

[10] Figure 2.2 of Storesletten shows that at age 20,25, 30,35, 40,45, the wage rate of the immigrant is lower than native by 15%, 20%, 17.8%, 16.4%, 12% and 13% respectively. By averaging those rates, I obtained 15.7%.

balanced growth path. In this paper, as a bench mark case, I assume that initial government debt or asset level is 0 percent of the private capital and I experiment with the assumption that the government debet is 10 percent or minus 10 percent of the private capital.

For the immigrant/native ratio(INR), I calculate the data from the US census at 2000. US Census 2000 shows that INR above the age of 20 is 15.5 percent. Thus, I assume that INR at the initial balanced growth path is 15.5 percent. For the target INR, I look at the past US data. Historically, INR was 5.5 percent in 1960 and increased to 11.1 percent in 2000. I assume that 10 percent point increase of INR is tolerable when it takes 100 years for the INR to reach the target level. Thus,I assume that the target INR is 25.5%. In the benchmark case, the INR starts to increase from 15.5 percent and reaches the target INR at 100th year. After 100th year, it remains on the same level. To see how the speed of increasing INR affects my results, I also consider other cases whether INR reaches the target INR at 75 year or 50th year. I assume that all immigrants come at the age of 1. The graph of INR over time is shown on Figure 4.

Once the age-nativity specific fertility rate, infant-child mortality and the initial immigrant/native ratio are set, then the annual population growth rate of immigrants and native are calculated automatically according to equation (48) with the steady state assumption on the population. With the estimated age-nativity specific fertility, the infant-child mortality and the initial immigrant-native ratio which is 15.5 percent, the annual growth rate of population of age 1 becomes 0.42 percent[11]. This implies that at the initial steady state, the government accepts immigrant so that the annual growth rate of immigrant of age 1 is becoming 0.42 percent. To check the sensitivity of my result regarding the initial INR, I also simulat the model with several other INR.

[11]The annual CPS data on immigrants and native from 1995 to 2010 shows that the median annual growth rate of the population(sum of natives and immigrants) aged from 20 to 40 is 0.12% while the medain growth rate of immigranted aged from 20 to 40 is 1.85%. Thus, the theoretically predicted growth rate of immigrant, 0.42%, is between the growth rate of immigrants aged 20 to 40 and the growth rate of the population aged 20-40.

4.3 Results

Figure 5, 6 and 7 show the life-cycle asset balance, consumption and leisure of an consumer at the initial balanced growth path in the benchmark cases which corresponds to row (1)-(18) Table 2. . At the age of 46, the consumption of leisure becomes 1 due to the mandatory retirement. At the initial balanced growth path, caital output ratio is 3.6 and it is higher than the standard level in the litarure (Cooly). This is partly due to setting the time preference rate higher than one. Thus, I condcut the robustness checks by lowering the time preference rate.

Table 2-4 shows the parameter values and the welfare effect of accepting immigrant. As mentioned in the above, the initial INR is set at 15.5 percent and the target INR is set 25.5 percent except Table 4 row (13)-(21). For the speed of accepting immigrant, I consider three cases where it takes 100 years, 75 years and 50 years for the INR to reach the target INR. This is shown in column (2). The column (3) of Table 2 shows the share of the government surplus for the government saving, which is V in equation (51). Column (4)-(8) are the value that are calculated within the simulation. Column (4) shows how many years it takes for the economy to reach the golden rule level. When it does not reach the golden rule within 300 years, then it is indicated with * or **. * indicates that the capital labor ratio at 300th year is higher than at the initial balanced growth path and it is increasing at 300th year but it does not reach at the 300th year ** indicates that capital labor ratio at 300th year is lower than at the initial balanced growth path. For example, in row (1), which is the benchmark case, it takes 111 years to reach the golden rule level. The column (5) shows how much capital stock per efficient unit of labor increases on the new balanced growth path compared with the initial balanced growth path. In the benchmark case, it shows that capital stock per efficient unit of labor increases 115 percent compared with the level at the initial balanced growth path. Column (6) shows that the publicly provided private goods will increase by about 50 percent in the benchmark case.To calculate column (5) ,(6) I evaluate at the year when the economy reached the golen rule level. When the economy does not reach the golden rule level, I evaluate at 300th year. the utility of the cohort who is born when the economy reach the golden rule. Column (7) shows how much percentage the utility, measured by the expenditure function, of the cohort who are born when the economy reach the golden rule the increases compared with the utility of the same cohort who would be on the initial balanced growth path. When the economy does not reach the golden rule, I calculate the ulity of the cohort who is born at 300th year. In the expenditure function, the price vector at the initial steady state is used for evaluating the utility Column (8) shows percentage increase of the sum of the present discounted value of the increased utility of all native and their descendant, measured by the expenditure function. Colum (8) measure the Pareto-improvement from the point of the native and thier descendants, which does not include immigrant and their descendants. To make the comparison easy, I use 5 percent discount rate to discount the life-time expenditure of the future cohorts. Column (9) quantify column (8) as the share of the initial GDP. Intuitively, it quantify the Pareto-improvement of accepting immigrant from the point of the natives and their descendants as the percentage of the initial GDP. For example, in row (1) in Table 2, it shows accepting immigrant brings Pareto-improvement whose present value is equal 19.76 percent of the initial GDP.

In row (2)-(3), I shorten the years used to reach the target INR and increase the speed of increasing INR. When the years used to reach the target INR is reduced to 50 years, instead of 100 years, the welfare gain of accepting immigrant amount to 29.82%.

In Table 2 row (4)-(18), I consider the case where the share of the surplus for the government saving, V in (51), is less than 100 percent. In those cases, the years taken to reach the golden rule level becomes longer as V becomes lower since the government saves less for future cohorts. On the other hand, the present discounted value of the increased utility, measured as the share of the initial GDP, becomes higher as V becomes lower as long as the V is greater or equal to 50% This is because in such cases, even cohort 1 will start to experience an increase of the utility through the increased publicly provided private goods. The present discount valued of the increased utility, measured as the share of the GDP becomes highest when $V = 50\%$. In this case, the net Pareto-improvement ranges from 28% to 43% of the initial GDP.

Figure 11 compares the utility level of all cohorts on the initial balanced growth path with the utility level of all cohorts with the increased immigrant/native ratio for different values of the share for the government saving(V). For example, when $V = 100\%$, all the surplus is used for the government saving until the economy reaches the golden rule level and the surplus is distributed to consumers only after the economy reaches the golden rule level. This implies that the utility of the cohort 110 who dies at 110th year starts to experience higher utility than at the initial balanced growth path.[12] In all cases considered in Table 2, 3 and 4, all cohorts are Pareto-improved and this confirms my theoretical results.

Figure 12 shows the marginal product of capital over time for different values of the share of the surplus for the governenet saving. V and when the length to reach the target INR is 100 years. When the government start to increase INR, the capital labor raito goes down and the marginal product of capital(MPK) starts to increase. However, the government uses some of the surplus for the government saving. As a result, for example when $V = 100$, the capital labor ratio start to increase and the MPK start to decrease around 30th year. MPK keeps decreasing until the economy reach the golden rule. As long as the share of the surplus for the government svaing is greater than 30%, MPK is lower than at the initial steady state and capital stock per efficient unit of labor is higher than at the initial balanced growth path.

4.4 Robustness Checks

In Table 3 and Table 4, I conducts robustness checks. Row (1)-(6) checks whether the result in Table 2 is sensitive with respect to the initial government debt(asset) level. As I argued in the previous sub-section, different authors assume different levels on the government debt(asset level) at the initial balanced growth path. In row (1)-(3) on Table 3, I set the initial government debt is 10 % of the private capital instead of 0 %. In row (4)-(6), I assume that the initial government debt level is -10% of the private capital.

Row (7)-(12) on Table 3 check whether the results in Table 2 are sensitive with respect to the replacement rate. The theoretical analysis shows that higher intergenerational redistribution implies higher welfare gain of accepting immigrants. Thus, it is predicted that as the replacement rate becomes lower,

[12] Reader might wonder why the 110th cohort, not 111th cohort, start to experience an increase of utility since the economy reaches the golden rule level of capital stock per capita at the 111th year. The 110th year, capital stock per capita is so close to the golden rule level. Thus, at 110th year, the government does not need to save so much for the government saving as in the previous periods. This implies that at 110th year, the government redistributes some of the surplus to the consumers.

the welfare gain of accepting immigrant becomes lower. Row (7)-(12) confirms this prediction. However, row (7)-(12) shows that the magnitude of the welfare gain does not change substantially.

Row (13)-(18) conduct sensitivity checks regarding on the earning level of the immigrants. Following Storesletten(1995), I set the immigrant wage rate to be 84.3 percent of the native's wage rate. In my calculation using CPS 2000 June supplement, I found that immigrant earning is 91 percent of native's earning. Row (13)-(15) assume that the wage rate of immigrant is 89.3 percent of the wage rate of native ubstead if 84.3 percent. Row (16)-(18) assumes that immigrant wage rate is 78.3 percent of the wage rate of the native. The results in row (13)-(18) show that the results in Table 2 practically do not change for those different earning levels of immigrants. For example, the welfare gain of Pareto-improvement changes from 19.20% to 20.95% or 18.55%.

Row (1)-(6) on Table 4 check whether the results on Table 2 change substantially by changing the parameter value of utility function. In Table 2 and Table 3, I assumed that CRRA is equal to 3. In row (1)-(6), I set CRRA to 2 or 4. The results in row (1)-(6) on Table 4 show that the results on Table 2 do not change substantially by changing CRRA.

Row (7)-(12) on Table 4 conducted sensitivity checks regarding the time preference rate. Following the literature, I set the time preference rate to 1.011. Although it is quite common to set the time presence greater than one in the computaitonal overlapping generation model, one might disagree such a value from the theoretical point. The time preference rate that is greater than one puts higher weight on future utility and this leads to higher saving rate. This is seen in the capital output ratio in the benchmark case. In the benchmark case (Table 2), capital output ratio on the initial balaned growth path is 3.6. This is a little bit higher than than the value used in the standard business cycle literature. In row (7)-(12) on Table 4, I recalcuate the results by setting the time preference less than or equal to one. This leads to lower saving and lower capital labor ratio and lower capital output ratio at the initial balanced growth path. When the time preference is equal to one, the capital output ratio is 3.26 and when the time preferance rate is 0.98, the caital output ratio is 2.7. Due to diminishing margianl product of capital, unfunded social security has a large negative effect when the capital labor ratio is low at the initial balanced growth path. This implies that the welfare gain of accepting immigrant will become higher. This is confirmed in row (7)-(12). For example, when the time presence rate is 0.98, the welfare gain of accepting immigrant is 29% of the initial GDP compared with 19 % in row (1) on Table 2.

Row (13)-(19) on Table 4 check how the results in Table 2 are affected by the initial INR. In Table 2, I assume that the itnial INR is 15.5 percent, which is the INR of aged above 20 in the US census 2000. On the other hand, using CPS data, INR keeps increasing in the last 20 years. In 1995, was 12.9 % but 2010, it was 18.3 percent. Thus, in row (13)-(15), I assume that the initial INR is 12.9% and the target INR is 22.9%. In row (16)-(19), I assume that the initial INR is 18.3% and the target INR is 28.3 percent. The results of row (13)-(19) shows that the effect of the initial INR is very minimum.

Row (19)-(24) condcut a sensitivity checks on the target INR. Although it seems torelable to increase INR by 10 percentage point in 100 years, in row (19)-(24) I lower the target immigrant/native ratio from 25.5 percent to 22.5 percent or 20.5 percent. The row (22) shows that even in the case that the government increases INR by 5 percentage point in 100 years, the Pareto-improving welfare gain of accepting immigrant amount to 9 percent of the initial GDP.

In row (25)-(27) on Table 4, I assume that the initial INR is 0% and the target INR is 10%. This is more consisten with the theoretical model since I have assumed that there is no immigrant at the initial steady state in the theoretical analysis. Row (25)-(27) shows that the result do not change practically compared with the results onTable 2.

5 Conclusion

In this paper, I have examined the welfare effect of accepting immigrants in the presence of a PYGO social security system qualitatively and quantitatively. First I have shown that if there are inter-generational transfers from the young to the old in the sense that the marginal product of labor of the young is higher than what the young receive while they are young, accepting immigrants is Pareto-improving. Second, I have shown that if the government adjusts the wage tax rate and the interest tax rate so that the after-tax wage rate and the interest rate tax rate after accepting immigrants are same as at the initially steady state, this Pareto-improvement is achieved. Third, I have shown that by accepting immigrants, it is possible to make the economy reach the golden rule level of capital stock per capita within a finite time in a Pareto-improving way.

Fourth, using the Auerbach-Kotlikoff computational dynamic overlapping generation model, I simulate years needed for the economy to reach the golden rule level of capital stock per capita and quantify the welfare gain of Pareto-improvement. The simulation results show the followings. (1) In all cases, all cohorts are Pareto-improved by increasing INR from 15.5 percent to 25.5 percent gradually for 100 years and this is consistent with the theoretical results. (2) The economy reaches the golden rule between 100-200 years if the share of the surplus for the government saving is greater or equal 70 percent. Even if capital labor ratio does not reach the golden rule level, it keeps increasing unless the share for the government saving is zero. (3) The welfare gain of those Pareto-improvement amounts to 19% to 43% of the initial GDP in the bench mark case(Table 2). (4)The results are robust for different values of several important parameters such as the repalcement rate, earning level of immigrants, initial government debt level, CRRA, time preference rate and initial INR. Those results indicate the welfare effect of accepting immigrnat is not trivial.

Appendix A1

Notice that in the programming problem, objective function is concave and the constrained set is convex. Thus, if some allocation satisfies the first order condition, it is also the solution of the programming problem. Now set up the Lagrangian function as follows:

$$
\begin{aligned}
L &= \frac{1}{1+\rho}[u^o(c_1^o) + v^o(g^o, q)] \\
&+ \sum_{t=1}^{\infty} \gamma_t \{U(c_t^y, c_{t+1}^o, g^y, g^o, g^{ind}, g^{ind}) - u^*\} \\
&+ \sum_{t=1}^{\infty} \lambda_t \{F(1+\pi_n, s_{t-1} + a_{t-1}) + (1-\delta)s_{t-1} \\
&\quad - (c_t^o + g^o + g^{ind}) - (1+\pi_n) \times (c_t^y + s_t + a_t + g^y + g^{ind}) \}
\end{aligned}
$$

The first order conditions are:

$$c_1^o : \frac{1}{1+\rho} u^{o\prime}(c_1^o) = \lambda_1; c_{t+1}^o : \gamma_t \frac{1}{1+\rho} u^{o\prime}(c_t^o) = \lambda_{t+1}; c_t^y : \gamma_t u^{y\prime}(c_1^y) = \lambda_t(1+n);$$

$$\gamma_t : U(c_t^y, c_{t+1}^o, g^y, g^o, q) - u^* = 0; a_t : \lambda_{t+1}\{\frac{\partial F}{\partial K} + 1 - \delta\} = \lambda_t(1+n)$$

$$\lambda_t : F(1+\pi_n, s_{t-1} + a_{t-1}) + (1-\delta)(s_{t-1} + a_{t-1}) - (c_t^o + g^o + q) - (1+\pi_n) \times (c_t^y + s_t + a_t + g^o + q)\ \} = 0$$

Now set the c_t^y, c_{t-1}^o, s_t, a_t, λ_t, γ_t as follows:

$$c_t^o = c^{o*}; c_t^y = c^{y*}; s_t = s^*; a_t = 0; \lambda_1 = \frac{1}{1+\rho} u^{o\prime}(c^{o*})$$

$$\lambda_{t+1} = \lambda_t \frac{1+\pi_n}{1+r^*}; \gamma_t \frac{1}{1+\rho} u^{o\prime}(c^{o*}) = \lambda_{t+1}$$

When we set c_t^y, c_{t+1}^o, s_t, a_t, λ_t, γ_t in this way, it clearly satisfies the first order conditions. Thus, the initial allocation is Pareto-efficient. Q.E.D.

Appendices B(appendices B are put on the journal's web page and author's webpage and , but are not attached to the main text in order to save the space.)

5.0.1 Appendix B1

The net government budget surplus at the period 1 is

$$\begin{aligned}
SP_1 &= w_1(l^n N_1^n + l^* N_1^m) - w^*(l^n N_1^n + l^* N_1^m)(1-\tau_w^*) \\
&+ r_1 s^* N_0 - r^* s^* N_0^n(1-t_r^*) - N_0^n(b + g^o + g^{ind}) - (N_1^n + N_1^m) \times (g^y + q) \\
&= w_1(l^n N_1^n + l^* N_1^m) - w^* l^* N_1^n(1-\tau_w^*) - w^* l^* N_1^m(1-\tau_w^*) \\
&+ r_1 s^* N_0^n - r^* s^* N_0(1-\tau_r^*) - N_0^n(b + g^o + g^{ind}) - (N_1^n + N_1^m)(g^y + g^{ind}) \\
&= w_1(l^* N_1^n + l^* N_1^m) - w^* l^* N_1^n((1-\tau_w^*) - w^* l^* N_1^m(1-\tau_w^*) \\
&+ r_1 s^* N_0^n - r^* s^* N_0^n(1-\tau_r^*) - N_0^n(b + g^o + g^{ind}) - (N_1^n + N_1^m)(g^y + g^{ind}) \qquad (58)
\end{aligned}$$

Note that from the budget constraint at the initial steady state, we have

$$\tau_w^* w^* N_1^n l^* + r^* s^* N_0^n \tau_r^* = N_0^n(b^* + g^o + g^{ind}) + N_1^n \times (g^y + g^{ind}) \qquad (59)$$

Thus, SP_1 becomes

$$SP_1 = w_1(l^*N_1^n + l^*N_1^m) - w^*l^*N_1^n - w^*l^*N_1^m(1-\tau_w^*)$$
$$+ r_1 s^* N_0^n - r^* s^* N_0^n - N_1^m(g^y + g^{ind})$$

Also, note that from the homogeneity of production function and Euler's theorem, we have

$$w_1(l^*N_1^n + l^*N_1^m) + r_1 s^* N_0^n = F((l^*N_1^n + l^*N_1^m),\ s^*N_0^n) - \delta s^* N_0^n$$
$$\text{and } w^*l^*N_1^n + r^*s^*N_0^n = F(N_1^n, s^*N_0^n) - \delta s^* N_0^n$$

Thus,

$$SP_1 = F(l^*N_1^n + l^*N_1^m,\ s^*N_0^n) - \delta s^*N_0^n - \{F(l^*N_1^n, s^*N_0^n) - \delta s^* N_0^n\} - w^*l^*N_1^m(1-\tau_w^*)$$
$$- N_1^m(g^y + g^{ind}) \tag{60}$$

Note that $N_1^m = \alpha N_1^n$ and $N_1^n = N_0^n(1+\pi_n)$. Thus,

$$SP_1 = N_0^n\{F(l^*(1+\pi_n)(1+\alpha),\ s^*) - F(l^*(1+\pi_n), s^*) - w^*l^*\alpha(1+\pi_n)(1-\tau_w^*)$$
$$-\alpha(1+\pi_n)(g^y + g^{ind})\} \tag{61}$$
$$= N_0^n \int_{l^*(1+\pi_n)}^{l^*(1+\pi_n)(1+\alpha)} [\frac{\partial F(z,s^*)}{\partial L} - w^*(1-t_w) - (g^y + g^{ind})\frac{1}{l^*}]dz \tag{62}$$

Noting $w^*(1-t_w)l^* = c^{y*} + s^*$, we have

$$SP_1 = \frac{N_0^n}{l^*}\int_{l^*(1+\pi_n)}^{l^*(1+\pi_n)(1+\alpha)} [l^*\frac{\partial F(z, s^*N_0^n)}{\partial L} - (c^{y*} + s^*) - (g^y + g^{ind})]dz$$

This is the equation (36).

5.0.2 Appendix B2

Note that

$$SP_2 = w_2\tau_{w2}(l^*N_2^n + l^*N_2^m) + r_2\tau_{r2}s^*(N_1^n + N_1^m) - (N_1^n + N_1^m)\times(b^* + g^o + g^{ind}) - (N_2^n + N_2^m)\times(g^y + g^{ind})$$
$$+ (1+r_2)a_1(N_1^n + N_1^m)$$

Using the definition of τ_{w2} and τ_{r2}, we have

$$
\begin{aligned}
SP_2 &= w_2(l^*N_2^n + l^*N_2^m) - w^*(l^*N_2^n + l^*N_2^m)(1-\tau_w^*) \\
&+ r_2 s^*(N_1^n + N_1^m) - r^*s^*(N_1^n + N_1^m)(1-\tau_r^*) - (N_1^n + N_1^m)(b + g^o + g^{ind}) \\
&- (N_2^n + N_2^m)(g^y + g^{ind}) + (1+r_2)a_1(N_1^n + N_1^m) \\
&= w_2(l^*N_2^n + l^*N_2^m) - w^*(l^*N_2^n + l^*N_2^m)(1-\tau_w^*) \\
&+ r_2 s^*(N_1^n + N_1^m) - r^*s^*(N_1^n + N_1^m)(1-\tau_r^*) - (N_1^n + N_1^m)(b + g^o + g^{ind}) \\
&- (N_1^n + N_1^m)(g^y + g^{ind}) + (1+r_2)a_1(N_1^n + N_1^m) \\
&= w_2(l^*N_2^n + l^*N_2^m) + r_2 s^*(N_1^n + N_1^m) - w^*(l^*N_2^n + l^*N_2^m) - r^*s^*(N_1^n + N_1^m) \\
&+ w^*(l^*N_2^n + l^*N_2^m)\tau_w^* + r^*s^*(N_1^n + N_1^m)\tau_r^* \\
&- (N_1^n + N_1^m)(b + g^o + g^{ind}) - (N_2^n + N_2^m)(g^y + g^{ind}) + (1+r_2)a_1(N_1^n + N_1^m)
\end{aligned}
$$

Note$N_1^m = \alpha N_1^n$ and $N_2^m = \alpha N_2^n$. Thus, we have

$$
\begin{aligned}
&= w_2(l^*N_2^n + l^*N_2^m) + r_2 s^*(N_1^n + N_1^m) - w^*(1+\alpha)l^*N_2^n - (1+\alpha)r^*s^*N_1^n \\
&+ (1+\alpha)w^*l^*N_2^n\tau_w^* + (1+\alpha)r^*s^*N_1^n\tau_r^* - (1+\alpha)N_1^n(b + g^o + g^{ind}) - (1+\alpha)N_2^n(g^y + g^{ind}) \\
&+ (1+r_2)a_1(N_1^n + N_1^m)
\end{aligned}
$$

Using $N_2^n = (1+\pi_n)N_1^n + (1+\pi_m)N_1^m$, we have

$$
\begin{aligned}
&w_2(l^*N_2^n + l^*N_2^m) + r_2 s^*(N_1^n + N_1^m) - w^*(1+\alpha)l^*\{N_1^n(1+\pi_n) + (1+\pi_m)N_1^m)\} - (1+\alpha)r^*s^*N_1^n \\
&+ (1+\alpha)w^*l^*\{N_1^n(1+\pi_n) + (1+\pi_m)N_1^m)\}\tau_w^* + (1+\alpha)r^*s^*N_1^n\tau_r^* - (1+\alpha)N_1^n(b + g^o + g^{ind}) \\
&- (1+\alpha)\{N_1^n(1+\pi_n) + (1+\pi_m)N_1^m)\}(g^y + g^{ind}) \\
&+ (1+r_2)a_1(N_1^n + N_1^m)
\end{aligned}
$$

Using the government budget constraint at the initial steady state, we have

$$(1+\alpha)w^*l^*\{N_1^n(1+\pi_n)\}\tau_w^* + (1+\alpha)r^*s^*N_1^n\tau_r^* - (1+\alpha)N_1^n(b+g^o+g^{ind}) - (1+\alpha)\{N_1^n(1+\pi_n)\}(g^y+g^{ind}) = 0$$

Thus, SP_2 becomes

$$w_2(l^*N_2^n + l^*N_2^m) + r_2 s^*(N_1^n + N_1^m) - w^*(1+\alpha)l^*\{N_1^n(1+\pi_n) + (1+\pi_m)N_1^m)\} - (1+\alpha)r^*s^*N_1^n$$
$$+ (1+\alpha)w^*l^*\{(1+\pi_m)N_1^m)\}\tau_w^* - (1+\alpha)\{(1+\pi_m)N_1^m)\}(g^y + g^{ind}) + (1+r_2)a_1(N_1^n + N_1^m)$$

$$= w_2(l^*N_2^n + l^*N_2^m) + r_2 s^*(N_1^n + N_1^m) - w^*(1+\alpha)l^*\{N_1^n(1+\pi_n) - (1+\alpha)r^*s^*N_1^n$$
$$- (1+\alpha)w^*l^*\{(1+\pi_m)N_1^m)\}(1-\tau_w^*) - (1+\alpha)\{(1+\pi_m)N_1^m)\}(g^y + g^{ind}) + (1+r_2)a_1(N_1^n + N_1^m)$$

From Euler's theorem, we have

$$w_2(l^*N_2^n + l^*N_2^m) + r_2(N_1^n + N_1^m)(s^* + a_1) = F(l^*N_2^n + l^*N_2^m, (N_1^n + N_1^m)(s^* + a_1)) - \delta(N_1^n + N_1^m)(s^* + a_1)$$
$$w^*l^*N_1^n(1+\pi_n) + r^*s^*N_1^n = F(l^*N_1^n(1+\pi_n),\ s^*N_1^n) - \delta s^*N_1^n$$

Thus, SP_2 becomes

$$SP_2 = F(l^*N_2^n + l^*N_2^m, (N_1^n + N_1^m)(s^* + a_1)) - \delta(N_1^n + N_1^m)(s^* + a_1) + (N_1^n + N_1^m)a_1$$
$$- (1+\alpha)\{F(l^*N_1^n(1+\pi_n),\ s^*N_1^n) - \delta s^*N_1^n\} - (1+\alpha)w^*l^*\{(1+\pi_m)N_1^m)\}(1-\tau_w^*) - (1+\alpha)\{(1+\pi_m)N_1^m)\}(g^y + g^{ind}$$

Notice that $(1+\alpha)\delta s^*N_1^n = \delta(N_1^n + N_1^m)s^*$. Subtracting and adding $F(l^*N_2^n + l^*N_2^m, (N_1^n + N_1^m)(s^*)$ SP_2 becomes

$$F(l^*N_2^n + l^*N_2^m, (N_1^n + N_1^m)(s^* + a_1)) + (1-\delta)(N_1^n + N_1^m)a_1 - F(l^*N_2^n + l^*N_2^m, (N_1^n + N_1^m)s^*)$$
$$F(l^*N_2^n + l^*N_2^m, (N_1^n + N_1^m)s^*) - (1+\alpha)F(l^*N_1^n(1+\pi_n),\ s^*N_1^n)$$
$$- (1+\alpha)w^*l^*\{(1+\pi_m)N_1^m)\}(1-\tau_w^*) - (1+\alpha)\{(1+\pi_m)N_1^m)\}(g^y + g^{ind})$$

Note that $F(l^*N_2^n + l^*N_2^m, (N_1^n + N_1^m)s^*) = N_1^n(1+\alpha)(F(l^*\frac{N_2^n}{N_1^n}, s^*)$.Thus, SP_2 becomes

$$SP_2 = F(l^*N_2^n + l^*N_2^m, (N_1^n + N_1^m)(s^* + a_1)) + (1-\delta)(N_1^n + N_1^m)a_1 - F(l^*N_2^n + l^*N_2^m, (N_1^n + N_1^m)(s^*)$$
$$+ (1+\alpha)N_1^n\{(F(l^*\frac{N_2^n}{N_1^n}, s^*) - F(l^*(1+\pi_n),\ s^*) - w^*l^*(1+\pi_m)\frac{N_1^m}{N_1^n}(1-\tau_w^*) - (1+\pi_m)\frac{N_1^m}{N_1^n}(g^y + g^{ind})\}$$

Since $\frac{N_1^m}{N_1^n} = \alpha$ and $N_2^n = N_1^n(1+\pi_n) + N_1^m(1+\pi_m)$ Thus SP$_2$

$$SP_2 = F(l^*N_2^n + l^*N_2^m, (N_1^n + N_1^m)(s^* + a_1)) + (1-\delta)(N_1^n + N_1^m)a_1 - F(l^*N_2^n + l^*N_2^m, (N_1^n + N_1^m)(s^*)$$

$$+ (1+\alpha)N_1^n\{(F(l^*((1+\pi_n) + (1+\pi_m)\alpha), s^*) - F(l^*(1+\pi_n),\ s^*) - w^*l^*(1+\pi_m)\alpha(1-\tau_w^*) - (1+\pi_m)\alpha(g^y + g^{ind})\}$$

$$= \int_{(N_1^n+N_1^m)s^*}^{,(N_1^n+N_1^m)(s^*+a_1)} \{\frac{\partial F(l^*N_2^n + l^*N_2^m, z)}{\partial K} + (1-\delta)\}dz$$

$$+ (1+\alpha)N_1^n \int_{l^*(1+\pi_n)}^{l^*\{(1+\pi_n)+(1+\pi_m)\alpha\}} \{\frac{\partial F(z, s^*)}{\partial L} - w^*(1-\tau_w^*) - (g^y + g^{ind})\frac{1}{l^*}\}dz$$

$$= \int_{(N_1^n+N_1^m)s^*}^{(N_1^n+N_1^m)(s^*+a_1)} \{\frac{\partial F(l^*N_2^n + l^*N_2^m, z)}{\partial K} + (1-\delta)\}dz$$

$$+ (1+\alpha)\frac{N_1^n}{l^*} \int_{l^*(1+\pi_n)}^{l^*\{(1+\pi_n)+(1+\pi_m)\alpha\}} \{l^*\frac{\partial F(z, s^*)}{\partial L} - w^*l^*(1-\tau_w^*) - (g^y + g^{ind})\}dz$$

Since $w^*l^*(1-\tau_w^*) = c^{y*} + s^*$, we have equation (39)

5.0.3 Appendix B3

As in the previous sub-section, consider the same tax adjustment.

$$(1-\tau_{wt})w_t = (1-\tau_w^*)w^* \text{ and } (1-\tau_{rt})r_t = (1-\tau_r^*)r^* \tag{63}$$

The government surplus at the period t is

$$SP_t = \tau_{wt}w_1l^*N_1^n + \tau_{wt}w_1\phi l^*N_1^m + \tau_{rt}r_ts^*N_{t-1}^n + \tau_{rt}r_ts^*N_{t-1}^n - (N_1^n + N_1^m) \times (g^y + g^{ind}) - N_0^n \times (g^o + g^{ind})$$

Adding and subtracting $\tau_{wt}w_1l^*N_1^m + +\tau_{rt}r_ts^*N_{t-1}^m - N_1^m \times (g^y + g^{ind}) - N_{t-1}^m \times (g^o + g^{ind})$, we have

$$SP_t = \tau_{wt}w_1l^*N_1^n + \tau_{wt}w_1l^*N_1^m + \tau_{rt}r_ts^*N_{t-1}^n + \tau_{rt}r_ts^*N_{t-1}^m - (N_1^n + N_1^m) \times (g^y + g^{ind}) - N_0^n \times (g^o + g^{ind})$$

$$\{\tau_{wt}w_1\phi l^*N_t^m + \tau_{rt}r_ts^mN_{t-1}^n - N_t^m(g^y + g^{ind}) - N_{t-1}^m(g^o + g^{ind})\}$$

$$- \{\tau_{wt}w_1l^*N_t^m + \tau_{rt}r_ts^nN_{t-1}^n - N_t^m(g^y + g^{ind}) - N_{t-1}^m(g^o + g^{ind})\}$$

Note that the first line of the above equation is equal to SP_t in the previous section. The sum of second line and the third line is equal to Redistribution$_t$. Thus, if (**??**) is satisfied, then(63) is feasible for all periods. The government can use the surplus for increasing publicly provided private goods. Therefore, Pareto-improvement is feasible.

5.0.4 Appendix B4

From the appendix B3, the government budget surplus at the period t can be rewritten as

$$SP_t = \text{Redistribution}_t \ + \int_{(N_{t-1}^n+N_{t-1}^m)s^*}^{(N_{t-1}^n+N_{t-1}^m)(s^*+a_{t-1})} \{\frac{\partial F(l^n N_t^n + l^n N_t^m, z)}{\partial K} + (1-\delta)\}dz \tag{64}$$

$$+ (1+\alpha)N_{t-1}^n \int_{l^n(1+\pi_n)}^{l^n((1+\pi_n)+(1+\pi_m)\alpha} \{\frac{\partial F(z(1+\alpha)N_{t-1}^n, s^* N_{t-1}^n(1+\alpha))}{\partial L} - w^*(1-\tau_w^*) - (g^y + g^{ind})\frac{1}{l^n}\}dz \tag{65}$$

Suppose that at each period $t \geq 2$,the government uses the surplus that arise from the second term. This implies that

$$a_t = \frac{1}{N_t^n + N_t^m} \int_{(N_{t-1}^n+N_{t-1}^m)s^*}^{(N_{t-1}^n+N_{t-1}^m)(s^*+a_{t-1})} \{\frac{\partial F(l^n N_t^n + l^n N_t^m, z)}{\partial K} + (1-\delta)\}dz$$

Then, the proof is the same as the proposition 3.

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Table 1 Number of birth at each age for native and immigrnt in the simulation		
age	number of birth of immigrant	number of birth of native
1	0.1934279	0.1743087
2	0.0520106	0.0474751
3	0.0550358	0.0505587
4	0.0569373	0.0527624
5	0.0578227	0.054141
6	0.0577998	0.0547487
7	0.0569765	0.0546402
8	0.0554604	0.0538698
9	0.0533594	0.052492
10	0.0507811	0.0505614
11	0.0478333	0.0481323
12	0.044624	0.0452592
13	0.0412605	0.0419967
14	0.0378511	0.0383992
15	0.0345031	0.034521
16	0.0313245	0.0304168
17	0.0284231	0.026141
18	0.0259064	0.0217481
19	0.0238824	0.0172925
20	0.0224589	0.0128286
21	0.0217435	0.0084112
22	0.0218439	0.0040944
23-80	0	0

Note

1. The calculaton is based on Figure 3. Let TB(i,j) be the vertical axis of group j of Figure 3 where j is native or immigrant. Then, the number of birth i of group j in the model is calcluated as follows. When i=1, the number of birth of age i of group j is TB(20,j)/2. When 2<=i<=22, the number of birth is (TB(19+i,j)-TB(18+i))/2.

Table 2 The effect of increasing immigrant ratio
The role of different share of the surplus for the government saving

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years to reach the target immigrant/ native ratio	share of the surplus for the gov. saving	years taken to reach goldren rule	% increase of capital stock per efficient unit labor at a golden rule	%change of publicly provided private goods per capita at golden rule	% change of welfare of the cohort born at a golden rule	% change of sum of present discounted value of welfare of all natives and their descendants	share of sum of present discounted value of welfare gain of all natives and their descendants in initial GDP
Bench mark case :share of the surplus for the gov. saving =100%								
1	100	100%	111	115.12%	52.73%	11.41%	0.38%	19.20%
2	75	100%	103	114.57%	52.24%	11.96%	0.47%	23.84%
3	50	100%	96	114.96%	53.76%	12.59%	0.59%	29.82%
share of the surplus for the gov. saving =90%								
4	100	90%	124	115.12%	52.73%	11.41%	0.43%	21.63%
5	75	90%	117	114.57%	52.24%	11.96%	0.53%	26.451%
6	50	90%	110	114.96%	53.76%	12.59%	0.66%	32.927%
share of the surplus for the gov. saving =70%								
7	100	70%	179	115.12%	52.73%	11.41%	0.52%	26.04%
8	75	70%	169	114.57%	52.24%	11.96%	0.64%	31.929%
9	50	70%	158	114.96%	53.76%	12.59%	0.80%	40.077%
share of the surplus for the gov. saving =50%								
10	100	50%	300*	85.47%	45.98%	9.03%	0.55%	27.60%
11	75	50%	300*	87.25%	46.50%	9.67%	0.68%	34.27%
12	50	50%	300*	89.02%	47.51%	10.32%	0.87%	43.47%
share of the surplus for the gov. saving =30%								
13	100	30%	300*	17.55%	26.35%	5.29%	0.50%	24.92%
14	75	30%	300*	17.82%	26.56%	5.66%	0.62%	31.11%
15	50	30%	300*	18.13%	27.33%	6.04%	0.79%	39.55%
share of the surplus for the gov. saving =0%								
16	100	0%	300**	-5.25%	10.23%	2.19%	0.42%	21.00%
17	75	0%	300**	-5.38%	10.08%	2.33%	0.52%	26.11%
18	50	0%	300**	-5.46%	10.63%	2.45%	0.66%	32.93%

Notes

1. In all rows, the initial immigrant/native ratio is 15.5% and target immigrant/native ratio is 25.5%. The replacement rate is 60 %, CRRA=3 and tht time preference rate is 1.011.

2. In all rows, the immigrant wage rate is 84.3 % of the native's wage rate.

3. * indicates that capital stock per efficinet unit labor does not reach the golden rule leve within 300 years, but its value at 300th year is higher than at the initial balanced growth path and keeps increasing at 300th year.

4. ** indicates that capital stock per efficinet unit labor does not reach the golden rule leve within 300 years and capital stock per efficin0et unit labor at 300th year is lower than at the initial balanced growth path.

Table 3: Robustness checks (1)

The role of initial government debt level, replacement rate and immigrant's earning level

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Case No.	years to reach the target immigrant/ native raito	initial governmen t debt ratio (% of priavate capital)	replacement ratio	years taken to reach goldren rule level	% increase of capital stock per efficient unit labor at the new balanced growth path	%change of publicly provided private goods per capita at the new balanced growth path	% change of welfare of the cohort born at the new balanced growth path	% change of present discounted value of welfare of all natives and their descendants	share of sum of present discounted value of welfare gain of all natives and their descendants in initial GDP
initial government debt ratio (% of private capital)= 10 % & replacement=0.6									
1	100	10%	0.6	105	127.55%	59.91%	12.91%	0.49%	23.02%
2	75	10%	0.6	98	126.96%	59.41%	13.52%	0.60%	28.46%
3	50	10%	0.6	90	127.42%	61.06%	14.23%	0.75%	35.50%
initial government debt ratio(% of private capital)= -10 % & replacement rate=0.6									
4	100	-10%	0.6	116	105.07%	46.99%	10.25%	0.31%	16.11%
5	75	-10%	0.6	109	104.55%	46.51%	10.73%	0.38%	20.09%
6	50	-10%	0.6	100	104.89%	47.94%	11.30%	0.48%	25.21%
initial government asset ratio(% of private capital)= 0% & replacement rate =0.55									
7	100	0%	0.55	113	109.89%	49.65%	10.77%	0.34%	17.43%
8	75	0%	0.55	105	109.36%	49.16%	11.28%	0.42%	21.69%
9	50	0%	0.55	98	109.72%	50.63%	11.87%	0.53%	27.17%
initial government asset ratio(% of private capital)= 0% & replacement rate =0.5									
10	100	0%	0.5	116	104.67%	46.59%	10.13%	0.30%	15.68%
11	75	0%	0.5	108	104.16%	46.11%	10.61%	0.369%	19.55%
12	50	0%	0.5	101	104.48%	47.52%	11.17%	0.463%	24.53%
human capital of immigrant is 89.3 % of the native's human capital									
13	100	0%	0.6	110	111.82%	51.41%	11.25%	0.41%	20.95%
14	75	0%	0.6	102	111.31%	51.00%	11.79%	0.511%	25.96%
15	50	0%	0.6	94	111.69%	52.37%	12.40%	0.637%	32.37%
human capital of immigrant is 79.3 % of the native's human capital									
16	100	0%	0.6	113	111.75%	50.34%	10.89%	0.36%	18.55%
17	75	0%	0.6	106	111.33%	49.91%	11.39%	0.445%	22.93%
18	50	0%	0.6	99	111.57%	51.05%	11.98%	0.557%	28.66%

Notes

1. In all rows, the initial immigrant/native ratio is 15.5% and target immigrant/native ratio is 25.5%. CRRA=3 and the time preference rate=1.011

2. The immigrant wage rate is 84.3 % of the native's wage rate in all rows except row (13)-row (18).

Table 4 : Robustness checks (2)

The role of CRRA and time preference, and the initial and target INR

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Row No.	years to reach the target immigrant/ native ratio	years taken to reach goldren rule	% increase of capital stock per efficient unit labor at a new s.s	%change of publicly provided private goods per capita at new s.s.	% change of welfare of the cohort born at a new s.s.	% change of sum of present discounted value of welfare of all natives and their descendants	share of sum of present discounted value of welfare gain of all natives and their descendants in initial GDP
CRRA=2 & time preference=1.011 & initial I NR=15.5% & target INR=25.5%							
1	100	116	70.48%	32.35%	7.98%	0.28%	16.65%
2	75	107	70.10%	32.00%	8.37%	0.36%	21.38%
3	50	98	70.39%	33.24%	8.79%	0.47%	27.36%
CRRA=4 & time preference=1.011 & initial INR=15.5% & target INR=25.5%							
4	100	106	154.58%	71.26%	14.18%	0.52%	23.31%
5	75	99	154.04%	70.77%	14.83%	0.62%	28.128%
6	50	92	154.36%	72.07%	15.61%	0.76%	34.406%
CRRA=3 & time preference=1 & initial INR=15.5% & target INR=25.5%							
7	100	105	156.35%	72.32%	14.35%	0.50%	22.51%
8	75	99	155.69%	71.79%	15.01%	0.61%	27.28%
9	50	91	156.12%	73.39%	15.81%	0.75%	33.472%
CRRA=3 & time preference=0.98 & initial INR=15.5% & target INR=25.5%							
10	100	94	251.90%	117.47%	20.71%	0.80%	28.56%
11	75	89	251.02%	116.84%	21.62%	0.94%	33.42%
12	50	83	251.54%	118.63%	22.79%	1.12%	39.75%
CRRA=3 & time preference=1.011 & initial INR=12.9% & target INR=22.9%							
13	100	109	119.72%	55.70%	12.22%	0.39%	19.35%
14	75	102	119.01%	55.10%	12.82%	0.49%	24.19%
15	50	94	119.55%	57.07%	13.52%	0.61%	30.35%
CRRA=3 & time preference=1.011 & initial INR=18.3% & target INR=28.3%							
16	100	113	110.54%	49.79%	10.63%	0.38%	19.05%
17	75	105	110.12%	49.40%	11.11%	0.46%	23.53%
18	50	98	110.40%	50.55%	11.68%	0.58%	29.32%
CRRA=3 & time preference=1.011 & initial INR=15.5% & target INR=22.5%							
19	100	121	116.79%	48.47%	8.65%	0.25%	12.94%
20	75	114	116.36%	48.09%	8.95%	0.31%	15.86%
21	50	107	116.66%	49.27%	9.30%	0.39%	19.76%
CRRA=3 & time preference=1.011 & initial INR=15.5% & target INR=20.5%							
22	100	130	120.27%	46.77%	7.31%	0.17%	8.88%
23	75	124	119.90%	46.45%	7.49%	0.21%	10.84%
24	50	117	120.17%	47.47%	7.71%	0.26%	13.55%
CRRA=3 & time preference=1.011 & initial INR=0% & target INR=10%							
25	100	100	149.48%	75.06%	17.52%	0.42%	20.28%
26	75	92	146.82%	73.27%	18.60%	0.55%	26.47%
27	50	84	149.74%	81.42%	19.87%	0.69%	33.50%

Notes

1. In all rows, the immigrant wage rate is 84.3 % of the native's wage rate; replacement is 0.6.

2. In all rows, the share for the government surplus is 100 % and the initial government debt level is 0% of the private caital.

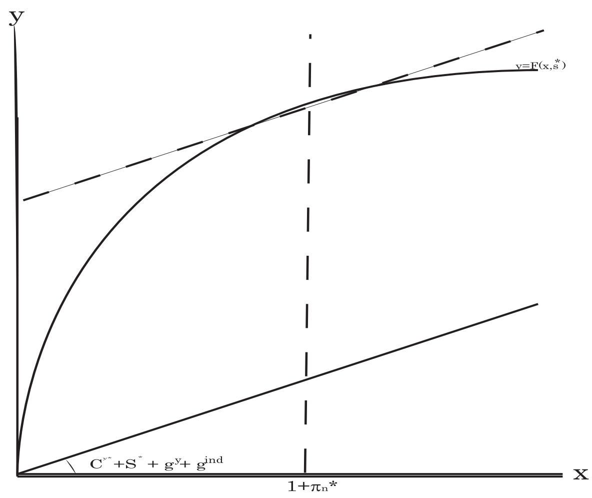

Figure 1: –The one plus population growth rate and the resource used for the one old person. The curve is $y = F(x, s^*)$ and the straight line is $y = (c^{y*} + s^* + g^y + g^ind) \times x$ where x is one plus the population growth rate. $y = F(x, s^*)$ measures the GDP per old and $y = (c^{y*} + s^* + g^y + g^ind) \times x$ measures the total resource used for young divided the number of old. The difference between curved line and the straight line measures the resource used per old at x. At $1 + \pi_n^*$, the resource for one old is maximized. If one plus the population growth rate is lower than $1 + \pi_n^*$, increasing the population growth rate will increase the resource available per one old without decreasing the resource used for young.

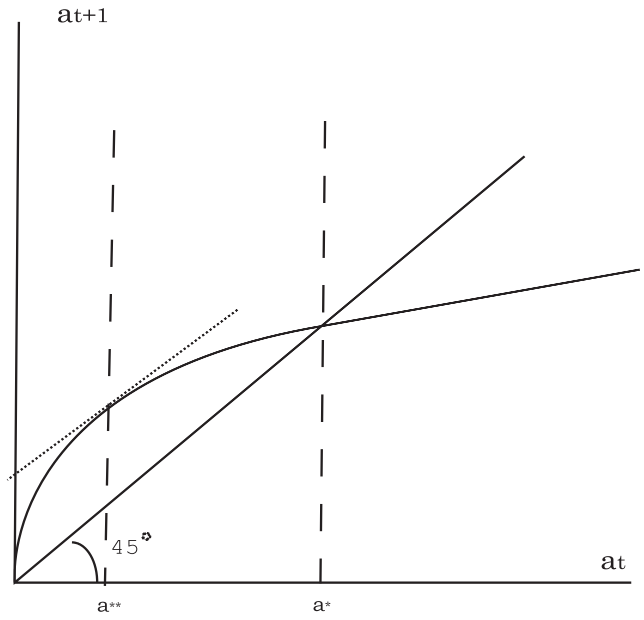

Figure 2: –The relationship between a_{t+1} and a_t. Starting from $a_1 > 0$, a_t will converge to a^*. This implies that before reaching a^*, a_t will reach a^{**}. At a^{**} the golden rule condition is satisfied.

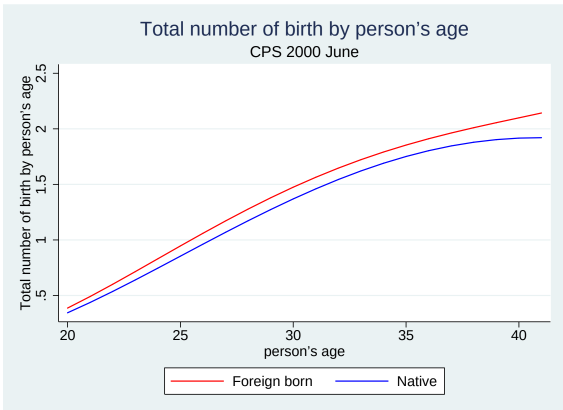

Figure 3: –Total number of births by each age. The data source is the CPS 2000 supplements. The total number of births by each age is regressed on the sixth order polynomial function of age separately for native and immigrant. The predicted values are plotted

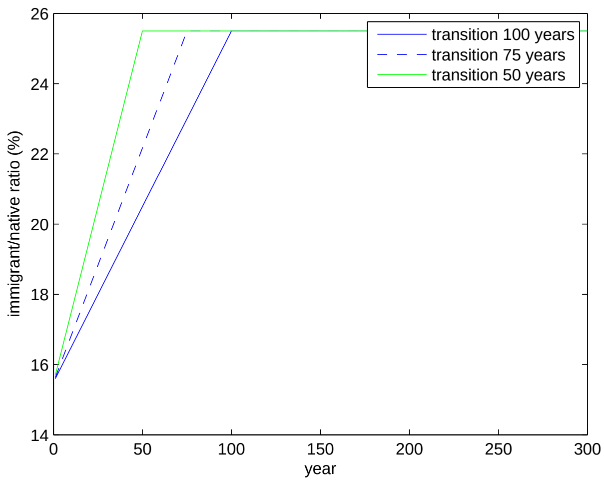

Figure 4: –Immigrant native ratio over time in the simulation. The vertical axis is the ratio of immigrant/naitve(percent).

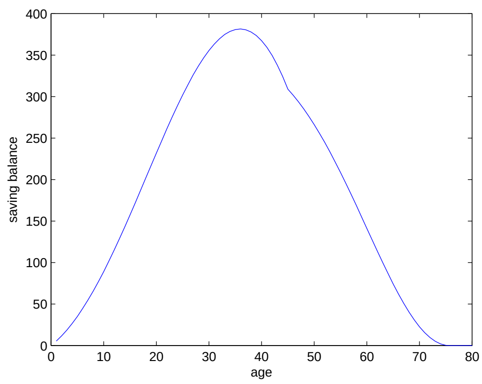

Figure 5: –Asset balance over life cycle at the initial balanced growth path

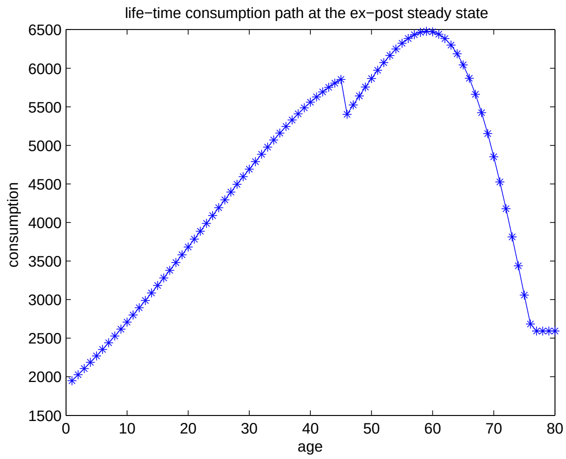

Figure 6: –Life cycle consumption path at the initial balanced growth path.

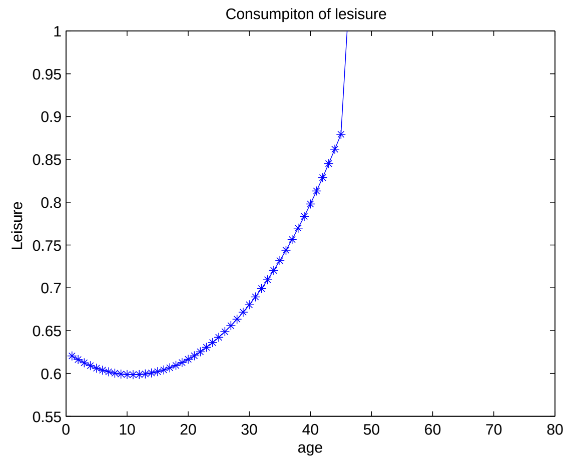

Figure 7: –Lifecycle consumption of leisure at the initial balanced growth path.

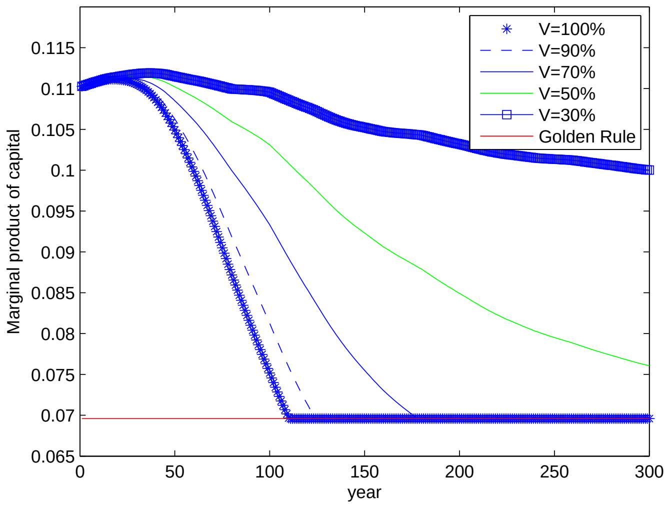

Figure 8: –The marginal product of capital over time for different values of the share of the surplus for the government saving. In those simulations, the years needed for INR to reach the target level is set 100 years.

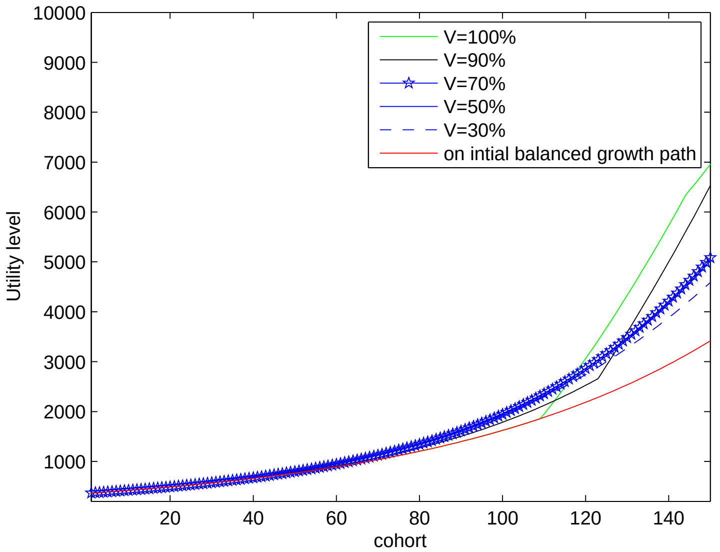

Figure 9: – Lifetime utility of all cohorts for different values of the share of the surplus for the government saving (V) and cohorts on the initial balanced growth path