Endogenous Human Capital Accumulation and Direct vs. Indirect Redistribution:Perfect Substitute Case

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Abstract

Recently, several papers analyzed the welfare effect of direct versus indirect redistribution when human capital accumulation is endogenous. Saez (2002) argued that indirect redistribution is not needed when human capital accumulation is endogenous, contrary to Naito's (1998) argument. However, recently Naito (2002) showed that when there are different types of human capital and high ability individuals have comparative advantage in accumulating skilled human capital, in the sense that the return from skilled human capital is higher for able individuals than less able individuals, the indirect redistribution is desired. But in Naito's discussion, he assumed that different types of human capital are imperfect substitutes in order to assume differentiability of optimal human capital function. This paper analyzes the direct versus indirect redistribution when different types of human capital are perfect substitutes and individuals accumulate only one type of human capital.

Keywords: Human capital accumulation, optimal taxation, non-linear income taxation and commercial policy
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1 Introduction

Recently, several papers analyzed the welfare effect of direct versus indirect redistribution when human capital accumulation is endogenous. Saez (2002) argued that when the human capital accumulation is endogenous, indirect redistribution is not needed in contrast to Naito (1998)'s argument. On the other hand, recently Naito (2002) showed that when people have different comparative advantage in accumulating different types of human capital in the sense that the return from skilled human capital is larger for more able individuals than for less able individuals, indirect redistribution is desired. But Naito's (2002) analysis has one shortcoming. In his analysis, he assumed that two types of human capital accumulation are imperfect substitutes in order to assume differentiability of human capital accumulation function. In reality, however, people normally accumulate only one type of human capital and as a result, the choice of human capital becomes discrete. The purpose of this paper is to analyze the welfare effect of direct versus indirect redistribution when human capital accumulation is endogenous and different types of human capital are perfect substitutes.

When different types of skills are perfect substitutes and able individuals have higher return from accumulating skilled human capital than less able individuals, individuals whose ability is higher than a certain threshold level will accumulate only skilled human capital and individuals whose ability is lower than that threshold level will accumulate unskilled human capital. As a result, individual choice of the type of skills becomes discrete. In such a situation, the government can change the return of skilled and unskilled human capital by affecting the production side of the economy. For example, imposing a tariff on unskilled human capital intensive goods in a small open economy will decrease the return from skilled human capital and increase the return from unskilled human capital. On the other hand, a nonlinear income tax system can distribute income more directly but it is subject to information constraints since the government cannot observe whether an individual accumulates skilled human capital and unskilled human capital. The question that this paper analyzes is whether an indirect redistribution policy will be needed and will increase the social welfare when the government can also use a more direct policy tool.

Besides the reason mentioned in our previous paper (Naito 2002), conducting a welfare analysis when individual behavior includes a discrete choice is interesting from the theoretical reason as well. In many important economic situations such as the choice of location to live, the choice of technology by firms and labor market participation, a decision making by consumers or firms include discrete choices. Until very recent, welfare analysis that includes discrete choices was rare. As far as the author knows, only Boadway and Kuff (2001) started to investigate this issue very recently. They analyzed an optimal taxation problem when some of individuals are bunched at the bottom. Another purpose of this paper is to contribute such a literature as well.

Other than the paper by Boadway and Kuff (2001), the paper that is closely related with our paper is Saez (2002). In this paper, he developed a model where each individual choose his human capital endogenously and analyzed direct versus indirect redistribution. He showed that when human capital accumulation is endogenous indirect redistribution is not needed.

2 The model

The economy is small and open and there are two output goods: good 1 and good 2. Good 1 is a skilled human capital intensive good and good 2 is an unskilled human capital intensive good. For factors used for production, we assume that there are two types of human capital in this economy: skilled human capital and unskilled human capital. For consumers, there is a continuum of agents. We assume that the utility function of agent i has the following form:

$$u(c_{1i}, c_{2i}) - a^s h_i^s - a^u h_i^u$$

where $u(c_{1i}, c_{2i})$ is strictly increasing with each argument and strictly concave. c_{1i} and c_{2i} are the consumption of good 1 and good 2 by agent i. h_i^s and h_i^u are the level of skilled and unskilled human capital of individual i. h_i^s and h_i^u can be interpreted as knowledge level, years of education, experience and training for each type of skill. The above utility function shows that accumulating two types of human capital are perfect substitutes.

As for heterogeneity among agents, we assume that different types of agents have different comparative advantage in accumulating skilled and unskilled human capital. We index the level of comparative advantage by i and we assume that i takes any value between one and two and that it is distributed with the density function n_i. More specifically, we assume that the earning of agent i is

$$\text{earning}_i = w^s \times i^2 \times h_i^s + w^u \times i \times h_i^u \tag{1}$$

where w^s and w^u are the return from one efficient unit of skilled and unskilled human capital, respectively. We assume that the labor supply is fixed and it is normalized to one. (1) means that when individual i accumulates h_i^s units of skilled human capital and h_i^u units of unskilled human capital, the efficient unit of skilled human capital and unskilled human capital are $i^2 \times h_i^s$ and $i \times h_i^u$ and the total return from skilled human capital and unskilled human capital are $w^s \times i^2 \times h_i^s$ and $w^u \times i \times h_i^s$, respectively. Thus, in (1) the index i is trying to capture the idea that an individual who has higher ability has more comparative advantage in accumulating skilled human capital than agents who have lower ability.

When two types of skill accumulation are perfect substitutes in the disutility function, the agent always solves the following constrained disutility minimization problem:

$$Z(R, w^s, w^u, i) \equiv \min a_s h_i^s + a_u h_i^u$$
$$\text{st } R = w_s \times i^2 \times h_i^s + w_u \times i \times h_i^u$$

where R is pre-tax income. In the above problem, for any $i > i^* \equiv (a_s/a_u) \times (w_u/w_s)$ only skilled human capital is accumulated and for any $i < i^*$ only unskilled human capital investment is accumulated. Thus, $Z(R, w^s, w^u)$ is

$$Z(R, w^s, w^u, i) = a^s(\frac{R}{i^2 w^s}) \text{ for } i \geq i^*$$
$$Z(R, w^s, w^u, i) = a^u(\frac{R}{i w^u}) \text{ for } i < i^*.$$

Let $X(R)$ be an after-tax income schedule that the government designs. Then, each agent chooses his best R to maximize $U(q, X(R)) - Z(R, w^s, w^u, i)$. Once R is chosen, an agent chooses his optimal skill type and accumulates human capital to generate pre-tax income R. As for the government objection, let $\widetilde{v}(i)$ be the maximized value given the schedule $X(R)$:

$$\widetilde{v}(i) \equiv \max_R U(q, X(R)) - Z(R, w^s, w^u, i).$$

Then, the purpose of the government is to design a schedule of $X(R)$ to maximize the following utilitarian social welfare function:

$$\int_1^2 \widetilde{v}(i) n_i di \ . \tag{2}$$

For the analysis of the optimal schedule of $X(R)$, we assume that the schedule of $X(R)$ is a continuous function. Although it is possible that the optimal schedule of $X(R)$ is not continuous, the tax schedules of almost of all developed countries are continuous. When $X(R)$ is a continuous function, it is straightforward to show that $\widetilde{v}(i)$ is continuous with respect to i from Berg (1969)'s the theory of maximum.

In addition, there is an interesting property on $\widetilde{v}(i)$ in the neighborhood of i^* that turns out to be crucial for our result. The following lemma shows that property of $\widetilde{v}(i)$.

Lemma 1 *When i increases, the graph of $\widetilde{v}(i)$ has a counter-clockwise kink at i^*.*

Proof. Let a graph of $\widetilde{v}(i)$ when all agents choose skilled human capital be $\widetilde{v}_s(i)$. Also, let the graph of $\widetilde{v}(i)$ when all agents chose unskilled human capital be $\widetilde{v}_u(i)$. By the definition, the graph of $\widetilde{v}(i)$ is the upper envelope of $\widetilde{v}_s(i)$and $\widetilde{v}_u(i)$ and i^* is the switching point from unskilled human capital to skilled human capital. This implies that there is a counter-clockwise kink at i^* (See also Figure 1). ■

As for prices, we normalize the producer price and the consumer price of good 1 to one. Let p_2, q_2 and p_2^* be the consumer price and the producer price and the international price of good 2, respectively. As the purpose of this section is to examine whether introducing production distortion can increase the social welfare or not, we consider imposing a tariff on good 2. Although a tariff introduces not only a production distortion but also a consumption distortion, the first order effect of consumption distortion on welfare can be ignored as we will demonstrate. Let σ be a size of a tariff on good 2. Then, we will have

$$p_2 = q_2 = p_2^* + \sigma. \tag{3}$$

As for the equations determining the returns from skilled and unskilled human capital, we use the same assumptions of our previous paper (Naito 2002). The model is the standard two sector Heckscher-Ohlin model. In this economy there are two sectors. Each sector uses both skilled and unskilled human capital. Sector 1 is a skilled human capital intensive sector and produces good 1. The sector 2 is an unskilled human capital intensive sector and produces good 2. Consumers (workers) are perfectly mobile between two sectors. When an agent who has h_i^s units of skilled human capital and h_i^u units of unskilled human capital work in sector k, it means that sector k uses $i^2 \times h_i^s$ units of skilled human capital and $i \times h_i^u$ units of unskilled human capital. Each sector behaves as a price taker and maximizes its profit. Let $F^k(H_k^s, H_k^u)$ be the production function in sector $k = 1, 2$ where H_k^s and H_k^u is the total amount of skilled human capital and unskilled human capital used in sector k. We assume that $F^k(H_k^s, H_k^u)$ exhibits constant returns to scale and it is concave with respect to both arguments. Let $c^k(w^s, w^u)$ be the cost function in sector k to produce one unit of output in sector k when the return of one efficient unit of skilled human capital and unskilled human capital are w^s and w^u, respectively. When both good 1 and good 2 are produced at the equilibrium, w^s and w^u are determined

$$1 = c_1(w^u, w^u) \text{ and } q_2 = c_2(w^s, w^u), \tag{4}$$

From the Stolper-Samuelson theorem, $\partial w^s/\partial q < 0$ and $\partial w^u/\partial q > 0$.

The output of both goods are determined from the following factor market equilibrium conditions:

$$\frac{\partial c_1}{\partial w^s} y^1 + \frac{\partial c_2}{\partial w^s} y^2 = H^s \equiv \int_1^2 i^2 \times h_i^s \times n_i di \tag{5}$$

$$\frac{\partial c_1}{\partial w^u} y^1 + \frac{\partial c_2}{\partial w^u} y^2 = H^u \equiv \int_1^2 i \times h_i^u \times n_i di \tag{6}$$

where H^1 and H^2 is the total amount of skilled and unskilled human capital in this economy.

Although the output of both goods can be calculated from equation (5), it is more useful to work on the production possibility frontier for analytical reasons. Let the production possibility frontier of this economy be $\Gamma(H^s, H^u)$. Since the production function is concave and factor intensity of two sectors are different, production possibility set is convex. Then, the output of good 1 and good 2 are determined as the solution of the following constrained maximization problem:

$$\max \ y_1 + q_2 y_2 \quad \text{s.t.} \quad (y_1, y_2) \in \Gamma(H^s, H^u) = 0 \tag{7}$$

Thus, we can think that the output of good 1 and good 2 are function of q_2, H^s and H^u. Let $Y(q, H_u, H_u)$ be the output function of good 2. At the optimum, the slope of production possibility set is equal to the relative producer price of good 2. Thus, we obtain $Y_q \equiv \partial Y/\partial q_2 > 0$. The Rybcyzynski theorem shows that $Y_{H^u} \equiv \partial Y/\partial H^u > 0$ and $Y_{H^s} \equiv \partial Y/\partial H^s < 0$.

Now consider the problem of designing a nonlinear income tax system. The revelation principle guarantees that without loss of generality we can focus on the incentive compatible revelation mechanism. This implies that the government first asks each agent to announce their own type j, and based on this announcement of j, the government asks the agent to earn R_j as pre-tax income and to give X_j as after-tax income. Given this schedule of R_j and X_j, define $v(i)$ and $\widehat{v}(j;i)$ as follows:

$$v(i) = \max_j U(q, X_j) - Z(R_j, w^s, w^u, i)$$
$$\widehat{v}(j;i) = U(q, X_j) - Z(R_j, w^s, w^u, i)$$

The incentive compatibility constraint implies that $v(i) \geq \widehat{v}(j;i)$ for all i and j. Assuming that $X(j)$ and $R(j)$ are both differentiable in $(1, i^*)$ and in $(i^*, 2)$, the first order condition of the incentive compatibility constraint for the agent i in $(1, i^*)$ and $(i^*, 2)$ is

$$\left.\frac{\partial \widehat{v}(j,i)}{\partial j}\right|_{j=i} = \frac{\partial U}{\partial x}\frac{\partial x}{\partial j} - \frac{\partial Z}{\partial R}\frac{\partial R}{\partial j} = 0. \tag{8}$$

By using (8), we can calculate $dv(i)/di$ for i in $(1, i^*)$ and $(i^*, 2)$.

$$\frac{dv}{di} = 2a^s \frac{R_i}{i^4 w^s} i = 2a^s \frac{R_i}{i^3 w^s} \text{ for } i \in (i^*, 2) \tag{9}$$
$$\frac{dv}{di} = a^u \left(\frac{R_i}{i^2 w^u}\right) \text{ for } i \in (1, i^*) \tag{10}$$

Next we will check a single crossing property of the utility function $U(q, X) -$

$Z(R, w^s, w^u, i)$. The marginal rate of substitution between X and R is

$$\begin{aligned}\text{MRS(R,x)} &= \frac{1}{U_x}\frac{a^s}{i^2 w^s} \text{ for } i \in (i^*, 2) \\ &= \frac{1}{U_x}\frac{a^u}{i w^u} \text{ for } i \in (1, i^*)\end{aligned}$$

Thus the MRS(R,X) is a decreasing function of i and a single crossing property is satisfied. This means that the local incentive compatibility and the monotone condition of R are sufficient conditions for the global incentive compatibility (Fudenberg and Tirole, 1991). Also, the global incentive compatibility implies the local incentive compatibility. The local incentive compatibility and a single crossing property implies the monotonicity of X and R. Thus, (9), (10) and the monotonicity of R_i are necessary and sufficient condition for the global incentive compatibility constraint. Because of (9) and (10), as Mirrlees (1971) demonstrated, it is useful to think that the government will control $v(i)$ and R_i and that X_i is defined from the following relationship:

$$v(i) = u(q, x) - z(R_j, w^s, w^u, i)$$

Finally for analytical convenience, define w_i^s and h_i^s as $w_i^s \equiv i^2 w^s$ and $h_i^s \equiv R_i^s / w_i^s$. Similarly, define w_i^u and h_i^u as $w_i^u \equiv i w^u$ and $h_i^u \equiv R_i^u / w_i^u$. h_i^s and h_i^u are the nominal units of skilled and unskilled human capital. w_i^s and w_i^u are the net return of skilled and unskilled human capital. Then, (9) and (10) can be re-written as

$$\dot{v}^s = 2a^s h_i^s \times (1/i)$$

$$\dot{V}^u = a^u h_i^u \times (1/i).$$

Based on the setup, the purpose of the government is considered to solve the following programming problem:

$$W(\sigma) = \max \int_{i^*}^{2} v^s(i) n_i di + \int_{1}^{i^*} v^u(i) n_i di$$

$$st.$$

$$\dot{v}^s = 2a^s h_i^s \text{ for } i^* < i \leq 2 \qquad \text{(IC1)}$$

$$\dot{v}^u = a^u h_i^u \text{ for } 1 < i < i^* \qquad \text{(IC2)}$$

$$v^s(i^*) = v^u(i^*) \qquad \text{(BD)}$$

$$\int_{i^*}^{2} \{R_i^s - x(R_i^s, v_i^s, q, w^s, w^u, i)\} n_i di$$

$$+ \int_{1}^{i^*} \{R_i^u - x(R_i^u, q, v_i^u, w^s, w^u, i)\} n_i di$$

$$+ \sigma \{ \int_{1}^{2} n_i c_{2i} di - Y(p_2^* + \sigma, H^s, H^u) \} \geq 0 \qquad \text{(RC)}$$

$$\dot{R^s}_i \geq 0 \qquad \text{(MON1)}$$

$$\dot{R^u}_i \geq 0 \qquad \text{(MON2)}$$

$$R_{i^*}^s \geq R_{i^*}^u \qquad \text{(MON3)}$$

where

$$H^s = \int_{i^*}^{2} h_i^s \times i^2 \times n_i di \text{ and } H^u = \int_{1}^{i^*} h_i^u n_i di$$

The above programming problem deserves several comments. First, (IC1) and (IC2) are the local incentive compatibility constraints. Second, (BD) comes from the assumption that the tax schedule that the government designs is continuous and, as a result, the utility level of the agents must be continuous. Third, (RC) is the government budget constraint and (MON1), (MON2) and (MON3) are the monotonicity constraints. Now let μ_i^s,μ_i^u and λ be the Lagrangian multiplier of (IC1),(IC2) and (RC). Let β_1, β_2 ,β_3 and β_4 be the Lagrangian multiplier of (BD), (MON1), (MON2) and (MON3). The first order conditions can be calculated and we write in the Appendix to save the space. Then, what we need to know is the effect of increasing σ from zero on the social welfare and it is equal to $dW/d\sigma$. However, to calcluate $dW/d\sigma$, we assume the mononotonicy constrraint is not bindind at i^*. This assumption is equivalent to assuming that there is no bunching at i^*. This non-bunching condition is often assumed in the prevous papers. (Konishi 1994, Naito 1998). Finally to calcuate $dW/d\sigma$, by using the envelope theorem, we have (See Appendix)

$$\left.\frac{dW}{d\sigma}\right|_{\sigma=0} = \frac{\partial i^*}{\partial \sigma}\left\{\mu^s_{i^*} 2a^s h^s_i(1/i) - \mu^u_{i^*} a^u h^u_i(1/i)\right\} \tag{11}$$

From the FOC of $v^s_{i^*}$ and $v^u_{i^*}$, we have $\mu^s_{i^*} = \mu^u_{i^*}$. In addition, as we show in the Appendix μ^s_i and μ^u_i are always non-negative. Furthermore $a^s h^s_i(1/i)$ and $a^u h^u_i(1/i)$ are the right hand slope of v^s_i and the left hand slope v^u_i at i^*. From Lemma 1, the slope of v^s_i is steeper than the slope of v^u_i at i^*. Since $\frac{\partial i^*}{\partial \sigma} > 0$, we have $dW/d\sigma > 0$.

Proposition 1 *Consider a small open economy where individuals accumulate human capital endogenously and different types of human capital are perfect substitutes. Suppose that the social planner designs a nonlinear income tax to maximize the utilitarian social welfare function without any production distortion and there is no-bunching at the swithcing point i^*. Then, introducing a tariff on unskilled human capital intensive good will increase the social welfare.*

At this point, it would be useful to consider the economic meaning of (11). Figure 1 shows the graph of $v(i)$, $\hat{v}^s(i)$ and $\hat{v}^u(i)$. When the government increases the tariff σ from zero, the graph of $v^s(i)$ will shift downward and the graph of $v^u(i)$ shifts upward. As a result, i^* will increase. Also, notice that from (IC1) and (IC2), the slope of $\hat{v}^s(i)$ increases and the slope of $\hat{v}^u(i)$ decreases.

In the mechanism designs problem, $\dot{v}$, the slope of the value function, is related with how the compensation schedule must be sensitive with unobserved ability. When $\dot{v}$ is higher, it means that the social planner needs to give higher utility to those with higher ability. With redistributive social welfare function, the social planner wants to give higher utility to agents with lower ability. Thus, when $\dot{v}$ is high, the social planner needs to give higher levels of utility to agents with high ability when the social planner wants to give higher utility to those with lower ability. But the government budget must be balanced. Thus, the level of utility that the social planner can give to the agents with lower ability is limited when $\dot{v}$ is higher. In such a situation, if the government can makes $\dot{v}$ smaller exogenously, it is possible to increase the social welfare.

When σ increases, the change of $\dot{v}$ is not the same for all individuals however. As Figure 1 shows, all individuals whose own ability are lower than i^* will experience a decrease of $\dot{v}$ and all individuals whose own ability are greater than i^* and except the neighborhood of i^* will experience an increase of $\dot{v}$. But, as the analysis in the Appendix shows, the effect of a change of $\dot{v}$ for those agents is of the second order and can be replicated by the adjustment of the nonlinear income system. On the other hand, there are some individuals who experience the first order change of $\dot{v}$. Individuals whose ability is in $(i^*,\ i^* + \partial\iota^*/\partial\sigma)$ will switch from accumulating skilled human capital to unskilled human capital. Since the graph of $v(i)$ has a counter-clockwise kink at i^*, individuals in $(i^*, i^* + \partial\iota^*/\partial\sigma)$ will experience the first order decrease of $\dot{v}$. This implies that the government needs less ability-sensitive compensation schedule for those agents. Because this change of $\dot{v}$ has the first order effect, it will increase the social welfare.

(11) can be interpreted in terms of the marginal tax schedule as well. It is well-known in the optimal taxation literature that $\dot{v}(i)$ is related with the degree of distortion and the marginal tax rate of individual i. Thus, $\mu^s_{i^*}\{2a^s h^s_i(1/i) - a^u h^u_i(1/i)\}$ approximately measures a change of the marginal tax rate for those who switched from accumulating skilled human capital to skilled human capital. Since the marginal tax rate of those who accumulated skilled human capital is higher than

the marginal tax rate for those who accumulated unskilled human capital around i^*, the marginal tax rate will decrease.[1] Since this change of the marginal tax rate is of the first order, it can increase the social welfare.

3 Conclusion

This paper analyzed the effect of direct versus indirect redistribution on welfare in a model where individuals first choose the type of skills based on their comparative advantage and then accumulate the chosen type of human capital. In this situation, indirect redistribution has an effect that a nonlinear income tax cannot have. In many realistic situations, the government cannot observe and verify the type of human capital that each individual accumulates but it can observe earned income. On the other hand, indirect redistribution such as tariffs that affect the return from skilled and unskilled human capital can discriminate those individuals who accumulate different types of human capital. If low ability agents have comparative advantage in accumulating unskilled human capital and high ability agent have comparative advantage in accumulating skilled human capital and if the social planner is interested in redistributing from high ability individuals to low ability individuals, then indirect redistribution can have that desired effect that a nonlinear tax alone cannot have.

[1] Readers still might wonder why the marginal tax rate for those who accumulated skilled human capital is higher than those who accumulated unskilled human capital around i^*. The reason is around the right hand side of i^*, the marginal return from ability is higher at the right hand side of i^* than at the left hand side of i^* because i^* is a switching point.

Appendix

The Lagrangian is:

$$L = \int_1^{i^*} v^u(i)n_i di + \int_{i^*}^2 v^s(i)n_i di + \int_{i^*}^2 \mu_i^s\{\dot{v}^s - 2a^s h_i^s(1/i)\}di + \int_1^{i^*} \mu_i^u\{\dot{v}^u - a^u h_i^u(1/i)\}di$$
$$+\beta_1\{v_{i^*}^s - v_{i^*}^u\} + \int_1^{i^*} \beta_{3i}\dot{R^u}_i di + \int_{i^*}^2 \beta_{4i}\dot{R^s}_i di + \beta_5\{R_{i^*}^s - R_{i^*}^u\}$$
$$+\lambda\int_1^{i^*}\{R_i^u - x(R_i^u, v_i^u, q, w^s, w^u, i)\}n_i di + \lambda\int_{i^*}^2\{R_i^s - x(R_i^s, v_i^s, w^s, w^u, i)\}n_i di$$
$$+\lambda\sigma\{\int_1^2 n_i c_{2i} di - Y(p_2^* + \sigma, H^s, H^u)\}$$

By using the integral by parts, we obtain

$$L = \int_1^{i^*} v^u(i)n_i di + \int_{i^*}^2 v^s(i)n_i di + \mu_2^s v_2^s - \mu_{i^*}^s v_{i^*}^s - \int_{i^*}^2 \dot{\mu}_i^s v_i^s di$$
$$- \int_{i^*}^2 \mu_i^s 2a^s h_i^s(1/i)di + \mu_{i^*}^u v_{i^*}^u - \mu_1^u v_1^u - \int_1^{i*} \dot{\mu}_i^u v_i^u di - \int_1^{i^*} \mu_i^u a^u h_i^u(1/i)di$$
$$+ \beta_1\{v_{i^*}^s - v_{i^*}^u\} + \beta_{3i^*}R_{i^*}^u - \beta_{3,1}R_1^u - \int_1^{i^*} \dot{\beta}_{3i}R_i^u di$$
$$+ \beta_{4,2}R_2^s - \beta_{4i^*}R_{i^*}^s - \int_{i^*}^3 \dot{\beta}_{4i}R_i^s di + \beta_5\{R_{i^*}^s - R_{i^*}^u\}$$
$$+ \lambda\int_1^{i^*}\{R_i^u - x(R_i^u, v_i^u, q, w^s, w^u, i)\}n_i di + \lambda\int_{i^*}^2\{R_i^s - x(R_i^s, v_i^s, w^s, w^u, i)\}n_i di$$
$$+ \lambda\sigma\{\int_1^2 n_i c_{2i} di - Y(p_2^* + \sigma, H^s, H^u)\}$$

The first order condition for v_i^s, v_2^s, $v_{i^*}^s$, R_i^s, $R_{i^*}^s$, v_i^u, $v_{i^*}^u$, v_1^u, R_i^u and $R_{i^*}^u$ are

$$v_i^s : n_i - \dot{\mu}_i^s - \lambda n_i \frac{\partial x}{\partial v_i^s} + \lambda\sigma\frac{\partial c_{2i}}{\partial x_i}\frac{\partial x_i}{\partial v_i^s} = 0$$

$$v_2^s : \mu_2^s = 0$$

$$v_{i^*}^s : -\mu_{i^*}^s + \beta_1 = 0$$

$$R_i^s : -\mu_i^s \times 2a^s\frac{\partial h_i^s}{\partial R_i^s}(1/i) + \lambda n_i - \lambda n_i\frac{\partial x}{\partial R_i^s} + \lambda\sigma n_i\frac{\partial c_{2i}}{\partial x_i}\frac{\partial x_i}{\partial R_i} - \dot{\beta}_{4i} = 0$$

$$R_{i^*}^s : \beta_{4i^*} = \beta_5$$

$$R_1^s : \beta_{4,2} = 0$$

$$v_i^u : n_i - \dot{\mu}_i^s - \lambda n_i \frac{\partial x}{\partial v_i^s} + \lambda\sigma \frac{\partial c_{2i}}{\partial x_i}\frac{\partial x_i}{\partial v_i^s} = 0$$

$$v_{i^*}^u : \mu_{i^*}^u - \beta_1 = 0$$

$$v_1^u : \mu_1^u = 0$$

$$R_i^u : -\mu_i^u \times a^u \frac{\partial h_i^u}{\partial R_i^u}(1/i) + \lambda n_i - \lambda n_i \frac{\partial x}{\partial R_i^u} + \lambda\sigma n_i \frac{\partial c_{2i}}{\partial x_i}\frac{\partial x_i}{\partial R_i^u} - \dot{\beta}_{3i} = 0$$

$$R_{i^*}^u : \beta_{3i^*} = \beta_5$$

$$R_1^u : \beta_{3,1} = 0$$

Now we characterize those first order conditions. First, note that

$$\mu_i^s = \mu_{i^*}^s + \int_1^{i^*} n_j(1 - \lambda\frac{\partial x}{\partial v_j})dj \text{ and } \mu_{i^*}^u = \mu_1^u + \int_1^{i^*} n_j(1-\lambda\frac{\partial x}{\partial v_j})dj \qquad (12)$$

Thus, since $\mu_1^u = 0$, $\mu_i^s = \int_1^2 n_j(1-\lambda\frac{\partial x}{\partial v_j})dj$. Note that $\frac{\partial x}{\partial v_j} = 1/(U_x)$. A single crossing property and the monotonicity of R_i^s and R_i^u guarantees that x_i is increasing. This implies that $\frac{\partial x}{\partial v_j}$ is increasing and the inside of the integral is a decreasing function of i. Since $\mu_2^s = 0$, μ_i^u and μ_i^s are non-negative.

Now we examine $dW/d\sigma$ and evaluate at $\sigma = 0$. From the envelope theorem,

$$\begin{aligned}\frac{dW}{d\sigma} = \frac{\partial i^*}{\partial \sigma}\Big\{ & -v^s(i^*)n_{i^*} + v^u(i^*)n_{i^*} - \dot{\mu}^s{}_{i^*}\, v(i^*) - \mu_{i^*}^s \dot{v}^s{}_{i^*} \\ & + \dot{\mu}^s{}_{i^*} v(i^*) + \mu_{i^*}^s 2a^s h_i^s(1/i) + \dot{\mu}^u{}_{i^*} v^u(i^*) - \mu_{i^*}^u \dot{v}^u{}_{i^*} - \dot{\mu}^u{}_{i^*} v^u(i^*) \\ & - \mu_{i^*}^u a^u h_i^u(1/i) + \beta_1\{\dot{v}^s{}_{i^*} - \dot{v}^u{}_{i^*}\} + \beta_{3i^*}\dot{R}^u{}_{i^*} - \beta_{4i^*}\dot{R}^s{}_{i^*} - \lambda\{R_{i^*}^s - x(R_{i^*}^s, v_{i^*}^s, q, w^s, w^u, i^*)\}n_{i^*} \\ & + \lambda\{R_{i^*}^u - x(R_{i^*}^u, v_{i^*}^u, q, w^s, w^u, i^*)\}n_{i^*}\Big\} \\ & + \frac{\partial w^s}{\partial\sigma}\left\{-\int_{i^*}^2 \mu_i^s a^s \frac{\partial \overline{h}_i^s}{\partial w_i^s} i di - \lambda\int_{i^*}^2 \frac{\partial x}{\partial w^s} n_i di\right\} + \frac{\partial w^u}{\partial \sigma}\left\{-\int_{i^*}^2 \mu_i^u a^u \frac{\partial \overline{h}_i^u}{\partial w_i^u} di - \lambda \int_1^{i^*}\frac{\partial x}{\partial w^u} n_i di\right\} \\ & + \lambda\int_1^2 n_i c_{2i} di - \lambda Y(p_2^* + \sigma, H^s, H^u) + \int_{i^*}^2 -\frac{\partial x}{\partial q}\frac{\partial q}{\partial \sigma} n_i di - \frac{\partial x}{\partial q}\frac{\partial q}{\partial \sigma}\int_1^{i^*} n_i di\end{aligned}$$

From the Roy's identity, $c_{2i} = -\frac{\partial x}{\partial q}\frac{\partial q}{\partial \sigma}$. Thus,

$$\begin{aligned}\frac{dW}{d\sigma} = & \frac{\partial i^*}{\partial\sigma}\{\mu_{i^*}^s 2a^s h_i^s(1/i) - \mu_{i^*}^u a^u h_i^u(1/i)\} + \frac{\partial w^s}{\partial \sigma}\left\{-\int_{i^*}^2 \mu_i^s a^s \frac{\partial \overline{h}_i^s}{\partial w_i^s} i di - \lambda \int_{i^*}^2 \frac{\partial x}{\partial w^s} n_i di\right\} \\ & + \frac{\partial w^u}{\partial\sigma}\left\{-\int_{i^*}^2 \mu_i^u a^u \frac{\partial \overline{h}_i^u}{\partial w_i^u} di - \lambda\int_1^{i^*} \frac{\partial x}{\partial w^u} n_i di\right\} - \lambda Y(p_2^* + \sigma, H^s, H^u)\end{aligned}$$

Now we need to calculate the inside of the integral. Note that from the definition of h_i^s and h_i^u, we have

$$\frac{\partial h_i^s}{\partial w_i^s} = -h_i^s \frac{\partial h_i^s}{\partial R_i^s} \text{ and } \frac{\partial h_i^s}{\partial w_i^s} = -h_i^s \frac{\partial h_i^s}{\partial R_i^s}$$

This implies that

$$-\mu_i^s a^s \frac{\partial h_i^s}{\partial w_i^s} i = \mu_i^s a^s h_i^s \frac{\partial h_i^s}{\partial R_i^s} i \text{ and -}\mu_i^u a^u \frac{\partial h_i^u}{\partial w_i^u} = \mu_i^u a^u h_i^u \frac{\partial h_i^u}{\partial R_i^u}$$

By using the FOC of R_i^s and R_i^u,

$$\mu_i^s a^s h_i^s \frac{\partial h_i^s}{\partial R_i^s} i = h_i^s i^2 \{\lambda n_i - \lambda n_i \frac{\partial x}{\partial R_i^s} - \dot{\beta}_{4i}\}$$
$$\mu_i^u a^u h_i^u \frac{\partial h_i^u}{\partial R_i^u} = h_i^u i \{\lambda n_i - \lambda n_i \frac{\partial x}{\partial R_i^u} - \dot{\beta}_{3i}\}$$

Thus, $\frac{dW}{d\sigma}$ is

$$\begin{aligned}\frac{dW}{d\sigma} &= \frac{\partial i^*}{\partial \sigma} \{+\mu_{i^*}^s 2a^s h_i^s (1/i) - \mu_{i^*}^u a^u h_i^u (1/i)\} \\ &+ \frac{\partial w^s}{\partial \sigma} \left\{ \int_{i^*}^2 h_i^s i^2 \{\lambda n_i - \lambda n_i \frac{\partial x}{\partial R_i^s} - \dot{\beta}_{4i}\} di - \lambda \int_{i^*}^2 \frac{\partial x}{\partial w^s} n_i di \right\} \\ &+ \frac{\partial w^u}{\partial \sigma} \left\{ \int_{i^*}^2 h_i^u i \{\lambda n_i - \lambda n_i \frac{\partial x}{\partial R_i^u} - \dot{\beta}_{3i}\} di - \lambda \int_1^{i^*} \frac{\partial x}{\partial w^u} n_i di \right\} \\ &- \lambda Y(p_2^* + \sigma, H^s, H^u)\end{aligned}$$

Next, we need to calculate $\lambda \frac{\partial w^s}{\partial \sigma} \int_{i^*}^2 h_i^s i^2 n_i + \lambda \frac{\partial w^u}{\partial \sigma} \int_1^{i^*} h_i^u i n_i di$. Note that from the assumption on perfect competition, the following relationship always holds:

$$y_1 + (p_2^* + \sigma) y_2 = w^s \int_{i^*}^2 h_i^s i^2 n_i di + w^u \int_{i^*}^2 h_i^s i^2 n_i \tag{13}$$

In the above equation, $\int_{i^*}^2 h_i^s i^2 n_i di$ is the total efficient unit skilled human capital and $\int_{i^*}^2 i h_i^u i n_i di$ is the total efficiency unit unskilled human capital. Thus, the above equations says that the total revenue of this economy should be returned to consumers as labor income. Let $\pi(\sigma)$ be the total revenue of this economy. Then, $d\pi/d\sigma = \frac{\partial w^s}{\partial \sigma} \int_{i^*}^2 h_i^s i^2 n_i + \frac{\partial w^u}{\partial \sigma} \int_{i^*}^2 h_i^s i^2 n_i$. In other words, the change of the total earnings when the amount of human capital is fixed is equivalent to the change of the total revenue of this economy when the total amount of skilled and unskilled human capital are fixed. On the other hand, the total revenue of this economy is the solution for the following problem:

$$\pi(\sigma) = \max \ y_1 + q_2 y_2 \quad \text{s.t.} \quad (y_1, y_2) \in \Gamma(H^s, H^u) = 0$$

By using the envelope theorem, we have $d\pi/d\sigma = y_2$. Therefore,

$$\lambda y_2 = \lambda \frac{\partial w^s}{\partial \sigma} \int_{i^*}^{2} h_i^s i^2 n_i + \lambda \frac{\partial w^u}{\partial \sigma} \int_{1}^{i^*} h_i^u i n_i di.$$

Third, we will show that $h_i^s i^2 \frac{\partial x}{\partial R_i^s} = -\frac{\partial x}{\partial w^s}$ and $h_i^u i n_i \frac{\partial x}{\partial R_i^u} = -\frac{\partial x}{\partial w_i^u}$. From the definition of Z, we have

$$\frac{\partial Z}{\partial R_i^s} = a^s \frac{\partial h_i^s}{\partial R_i^s} \text{ and } \frac{\partial Z}{\partial w^s} = a^s \frac{\partial h_i^s}{\partial w_i^s} i^2 \text{ for } i \in (i^*, 2)$$
$$\frac{\partial Z}{\partial R_i^u} = a^u \frac{\partial h_i^u}{\partial R_i^u} \text{ and } \frac{\partial Z}{\partial w^s} = a^s \frac{\partial h_i^u}{\partial w_i^u} i \text{ for } i \in (1, i^*)$$

Thus, by using the definition of $\frac{\partial x}{\partial R_i^s}, \frac{\partial x}{\partial w^s}$,$\frac{\partial x}{\partial R_i^s}, \frac{\partial x}{\partial w^s}$, we can check that $h_i^s i^2 \frac{\partial x}{\partial R_i^s} = -\frac{\partial x}{\partial w^s}$ and $h_i^u i n_i \frac{\partial x}{\partial R_i^u} = -\frac{\partial x}{\partial w_i^u}$.

Finally, note that

$$-\int_{i^*}^{2} h_i^s i^2 \dot{\beta}_{4i} di = -\int \frac{R_i^s}{w^s} \dot{\beta}_{4i} di \text{ and } -\int_{1}^{i^*} h_i^u i \dot{\beta}_{3i} di = -\int \frac{R_i^u}{w^s} \dot{\beta}_{3i} di$$

By using integral by parts, we have

$$-\int_{i^*}^{2} h_i^s i^2 \dot{\beta}_{4i} di = \int_{1}^{i^*} \frac{\dot{R}_i^s}{w^s} \beta_{4i} di - \frac{R_2^s}{w^s} \beta_{4,2} + \frac{R_{i^*}^s}{w^s} \beta_{4,i^*}$$
$$-\int_{i^*}^{2} h_i^u i \dot{\beta}_{3i} di = \int_{1}^{i^*} \frac{\dot{R}_i^u}{w^u} \beta_{3i} di - \frac{R_2^u}{w^u} \beta_{3,1} + \frac{R_{i^*}^u}{w^u} \beta_{3,i^*}$$

Note that from the FOC of $R_{i^*}^s$, $R_{i^*}^u$ and the non-bunching assumption, $\beta_{3,i^*} = \beta_{4,i^*} = 0$. From the FOC of R_2^s and R_1^u , $\beta_{3,1} = \beta_{4,2} = 0$. From the complementary-slackness condition, $\frac{\dot{R}_i^s}{w^s}\beta_{4i} = 0$ and $\frac{\dot{R}_i^u}{w^u}\beta_{3i} = 0$. This implies that

$$-\frac{\partial w^s}{\partial \sigma} \int_{i^*}^{2} h_i^s i^2 \dot{\beta}_{4i} di - \frac{\partial w^u}{\partial \sigma} \int_{i^*}^{2} h_i^s i^2 \dot{\beta}_{3i} di = 0$$

Therefore, $dW/d\sigma$ is

$$\left.\frac{dW}{d\sigma}\right|_{\sigma=0} = \frac{\partial i^*}{\partial \sigma}\left\{\mu^s_{i^*} 2a^s h^s_i(1/i) - \mu^u_{i^*} a^u h^u_i(1/i)\right\}$$

From the FOC of $v^s_{i^*}$ and $v^u_{i^*}$, we have $\mu^s_{i^*} = \mu^u_{i^*}$. In addition, $a^s h^s_i(1/i)$ and $^u_{i^*} a^u h^u_i(1/i)$ are the right side slope of v^s_i and the left side slope v^u_i at i^* From Lemma 2, the slope of v^s_i is steeper than the slope of v^u_i at i^*. Since $\frac{\partial i^*}{\partial \sigma} > 0$, $\frac{dW}{d\sigma} > 0$.

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