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Restrictions in labor supply estimation: Is the MaCurdy critique correct?

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Abstract

According to the MaCurdy critique the Hausman method implicitly imposes restrictions that generate a positive Slutsky effect. This paper shows that the Hausman method imposes no other restrictions than alternative methods do and that the MaCurdy critique is unfounded.

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1. Introduction

There are two common methods to estimate labor supply functions, taking account of nonlinear budget constraints. In one method, often referred to as the Hausman method, one accounts for the complete form of the budget constraint and uses the maximum likelihood method to estimate the labor supply parameters. The other method linearizes budget constraints around observed points and uses some instrumental variables technique to break the correlation between measurement errors in observed hours, the net wage rate and the measure of nonlabor income. Pencavel (1986) mainly surveys studies using linearization techniques, whereas Hausman (1985) covers studies using the Hausman method. MaCurdy et al. (1990) and MaCurdy (1992) note that the two surveys present quite different views of the effect of tax reform on labor supply. In particular, the results presented in the Hausman survey in general imply larger substitution effects than the results given in Pencavel's survey. The aim of MaCurdy's papers is to reconcile these differences. MaCurdy claims that the differences arise because the Hausman method implicitly imposes restrictions that generate a positive Slutsky effect. In this paper I show MaCurdy's claim to be incorrect.

It is important to distinguish between the following two issues: (i) Under what conditions can estimated parameters be interpreted as labor supply parameters? (ii) Do any of the estimation procedures automatically produce parameter estimates that satisfy the Slutsky

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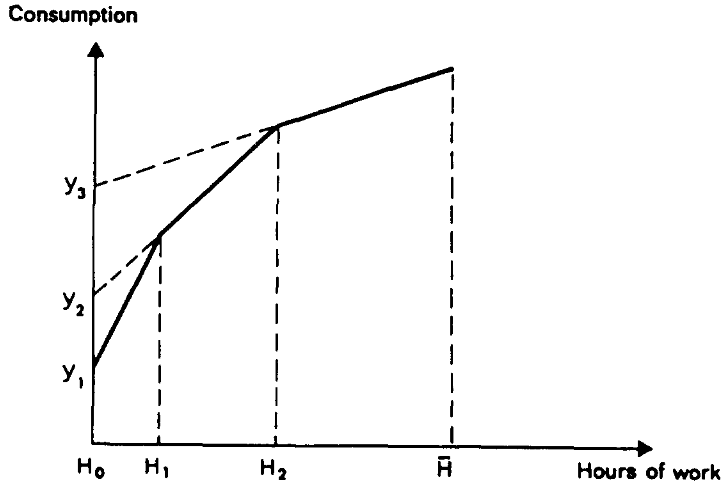

Fig. 1.

condition? As I show below, the answer to the first question is that the estimated parameters must be consistent with utility maximization with globally convex preferences. This is true whether we use the linearization method or the Hausman method. The answer to the second question is that no such implicit constraints exist for either method.

In Section 2 I describe the static labor supply model used in MaCurdy's papers. In Section 3 I briefly describe the linearization method and the Hausman method. Section 4 discusses the restrictions implied by each method. In Section 5 I discuss some empirical results and point to alternative reasons why results vary so much between studies. Section 6 summarizes.

2. The labor supply model

We assume that data are generated by utility maximization with globally strictly convex preferences, subject to the budget constraint, $C = Wh + Y - T(Wh, Y)$, where $C =$ consumption; $W =$ wage rate per hour; $h =$ hours of work; $Y =$ nonlabor income, and $T =$ taxes determined by the function $T(\cdot\,,\cdot)$. For simplicity we assume the tax-transfer system only consists of three income brackets generating a convex budget set of the form illustrated in Fig. 1. We denote the upper physical limit of hours of work by $\bar{H}$. H_j denotes the upper limit of the linear segment j. $\omega_j = W(1 - t_j)$ denotes the slope of the jth linear segment and y_j its intercept. In the following we will call y_j the virtual income for segment j.

Let individual i's utility function be $U(C_i, h_i, v_i)$, where v_i is an individual specific preference parameter. Maximization of the utility function subject to a linear budget constraint with slope ω_j and intercept y_j yields the basic supply function $f(\omega_j, y_j)$.[1] In empirical

[1] The concept of a basic supply function is introduced in Blomquist (1988). A basic supply function $f(\omega, y)$ shows desired hours of work generated by a *linear* budget constraint with slope ω and intercept y. The mongrel function shows desired hours of work as a function of the gross wage rate and before-tax nonlabor income, given a certain income tax function.

work a linear form is often used. The basic supply function will then have the form $h_j = f(\omega_j, y_j) = \mu + \alpha\omega_j + \beta y_j + v = \hat{h}_j + v$. Maximization of the utility function subject to the nonlinear budget constraint yields the mongrel function $h = m(W_i, Y_i, v_i)$. The functional form of $m(\cdot)$ depends both on the utility function and the tax function (see Blomquist, 1988). We assume there are also measurement and/or optimization errors. Thus, observed hours of work are given by $h_i^* = m(W_i, Y_i, v_i) + \epsilon_i$.

3. Two methods to estimate labor supply functions

3.1. The linearization method

Diewert (1971) noted that if we linearize the budget constraint around the utility optimum point, then the linearized budget constraint and the nonlinear budget constraint yield the same behavior. Thus, one way to proceed is to linearize individuals' budget constraints around the utility optimum points and use the slope and intercept as measures of the net wage rate and virtual income. Note that preferences must be globally strictly convex for the linearization method to be valid. If, for example, preferences are not globally convex, then the utility optimum for the linearized budget constraint might differ from the utility optimum for the nonlinear budget constraint. I want to emphasize that the linearization technique is based on the assumption that data are generated by utility maximization with globally convex preferences. I have never seen any other motivation for the linearization method.

Unfortunately, we usually do not know the utility optimum point, but only observed hours. This means that sometimes we will linearize the wrong segment, which will lead to measurement errors in the net wage rate and the nonlabor income. If OLS is applied to the data obtained by linearization a severe bias may result. To correct for this bias it is common practice to instrument the net wage rate and the virtual income.

3.2. The Hausman method

As for the linearization method, we assume data are generated by utility maximization with globally strictly convex preferences. We also assume that preferences are random and that there is a measurement error in hours of work. Given these assumptions, the log likelihood function is given by $\sum_i \log P(h_i^*)$, where i designates an observation and

$$P(h_i^*) = \underbrace{\int_{-\infty}^{\underline{v}_{1i}} b_1[h_i^* - H_0, v_i]\,\mathrm{d}v_i}_{\text{(lower limit)}} + \underbrace{\sum_{j-1}^{3} \int_{\underline{v}_{ji}}^{\bar{v}_{ji}} b_2[h_i^* - \hat{h}_{ji}, v_i]\,\mathrm{d}v_i}_{\text{(segment 1, 2, 3)}}$$
$$+ \underbrace{\sum_{j-1}^{2} \int_{\bar{v}_{ji}}^{\underline{v}_{j+1,i}} b_1[h_i^* - H_{ji}, v_i]\,\mathrm{d}v_i}_{\text{(kink 1, 2)}} + \underbrace{\int_{\bar{v}_{3i}}^{\infty} b_1[h_i^* - \bar{H}, v_i]\,\mathrm{d}v_i}_{\text{(upper limit)}}\,. \tag{1}$$

Expression (1) is reproduced from MaCurdy (1992) and corresponds to his eq. (4). The limits of integration are given by $\underline{v}_{ji} \equiv H_{j-1,i} - \hat{h}_{ji}$ and $\bar{v}_{ji} \equiv H_{ji} - \hat{h}_{ji}$; the function $b_1(\cdot\,,\cdot)$ is the bivariate density of (ϵ, v); and $b_2(\cdot\,,\cdot)$ is the joint density of $(v+\epsilon, v)$. The parameter estimates are obtained by finding the maximum of the log likelihood function.

Note that the form of the likelihood function follows from the assumption that preferences are globally strictly convex. If we specified another data-generating process, we would obtain a likelihood function that had a different form.

4. Restrictions

It is important to distinguish between two quite different types of restrictions. We can pose the question: Under what conditions can we interpret the estimates as labor supply parameters? A quite separate question is: Do any of the two estimation procedures automatically impose numerical constraints so that the estimated parameters always satisfy the Slutsky condition?

Both the linearization method and the Hausman method build on the assumption that data are generated by utility maximization with globally strictly convex preferences. This implies that we do not know how to interpret the results if the estimated parameters are not consistent with this assumption.[2] In particular, we cannot interpret the parameters as parameters in a labor supply model.

For both methods the parameter estimates are obtained by optimizing an objective function. It is true that we cannot interpret the results if the optimizing parameters are inconsistent with utility maximization. However, we can perform the optimization of the two objective functions without imposing the restriction that the parameter values should be consistent with utility maximization. Indeed, this is the normal way to proceed for both methods.

According to the MaCurdy critique, the Hausman method automatically imposes restrictions. As seen above we should distinguish between two quite different types of restrictions. Is the essence of the MaCurdy critique that we can only interpret the results as maximum likelihood estimates if the parameter values are consistent with utility maximization? Probably not, because it is a trivial comment. In all maximum likelihood estimation we specify a data-generating mechanism. If the estimated parameter values are inconsistent with this data-generating mechanism we do not know how to interpret the results. In fact, this is true for most estimation techniques. We specify a data-generating mechanism and design a method to estimate some parameters of this mechanism. If the parameters are inconsistent with the data-generating mechanism, we do not know how to interpret the results. This applies with equal force to the linearization technique as to the Hausman method.

Thus, I take the essence of the MaCurdy critique to be the assertion that we automatically impose constraints, guaranteeing a positive Slutsky effect, if the parameter estimates are obtained by maximizing an expression like $\sum_i \log P(h_i^*)$, with $P(h_i^*)$ being defined by (1). Is this assertion correct? A very easy way to show that it is not is to turn to MaCurdy's own empirical results. When MaCurdy et al. (1990) maximize an expression like $\sum_i \log P(h_i^*)$,

[2] There are several possible reasons for parameter values inconsistent with the utility maximization hypothesis. One is, of course, that data might not be generated by utility maximization. However, bad estimators, bad data or a bad choice of functional form for the basic labor supply function are other possibilities.

without imposing explicit constraints, they obtain parameter estimates inconsistent with utility maximization. In table 2 in MaCurdy et al. (1990) the authors report an estimated linear basic supply function with a wage rate coefficient of −79.2 and a nonlabor income coefficient of −0.0080. Evaluated at mean hours of work this implies a 'Slutsky term' of −61.31. In the working paper version of the 1990 paper (MaCurdy et al., 1988, p. 33) the authors comment on their results as: "In the case of the unconstrained results, estimates of the coefficients move to values that imply a negative Slutsky effect at all kinks." Thus, the empirical results in MaCurdy et al. (1990) clearly show that maximizing an expression like $\sum_i \log P(h_i^*)$ does not impose restrictions guaranteeing positive Slutsky effects.

MaCurdy focuses a lot on the parts of expression (1) that give the 'density contribution' from the kink points and claims that the Hausman method imposes constraints guaranteeing that these parts must be positive. Inspection of expression (1) shows that we very well can have a maximum of $\sum_i \log P(h_i^*)$ where the parts of $P(h_i^*)$ that correspond to kink points are negative. What is true is that at a maximum $P(h_i^*)$ must be positive for all i.[3] However, this only means that the *sum* of all the terms in (1) must be positive, not the single components. Since the terms which give the 'density contribution' from the linear segments always are positive, it leaves plenty of room for the 'kink terms' to be negative. The empirical results in MaCurdy et al. (1990) show that this indeed can occur in actual estimation.

MaCurdy's focus on the kink terms is strange, since these terms are not related to either of the two types of constraints that are of interest. Even if all kink point terms were positive, but preferences not globally strictly convex, we could not interpret $\sum_i \log P(h_i^*)$ as a likelihood function and the estimates as labor supply parameters. The crucial condition for the likelihood function to be well defined is not that the kink terms are positive, but that preferences are globally strictly convex. The numerical constraint implied by maximizing the *log* likelihood function is not related to the kink terms, but to the sum of all the terms in expression (1).[4]

5. Why do empirical results differ?

Studies using linearization techniques have often obtained results implying 'negative Slutsky effects'.[5] This means that the results are inconsistent with the basic assumption on which the

[3] The restriction that $P(h_i^*)$ must be positive is due to the fact that we use the *log* likelihood function. If we, instead, use the likelihood expression it is, in principle, possible to obtain an optimizing vector such that $P(h_i^*)$ is negative for some i. Of course, if this is the case the optimizing vector cannot be interpreted as a maximum likelihood estimate.

[4] If the likelihood function is maximized there are no numerical constraints whatsoever on the parameters. If the *log* likelihood function is maximized there are numerical constraints. Does this mean that we obtain biased results? Note that this is a question that concerns all maximum likelihood estimation. It is not a question particular to the Hausman method.

If the parameter vector that maximizes the likelihood function is consistent with the assumed data-generating mechanism, then the log likelihood function achieves its maximum for the same parameter vector. Hence, using the log likelihood function will, in this case not bias the results.

[5] Of course, by definition, there exist no negative Slutsky effects. What is meant here is that a mechanical application of the formula, which yields the Slutsky effect when data are generated by utility maximization, results in a negative value.

estimation technique builds and that we cannot interpret the results. In particular, we cannot interpret the parameter values as parameters of a labor supply model. One possible interpretation is that data refute the utility maximization hypothesis. However, other explanations are at hand. Simulation results in Blomquist (1992) show that if data are generated by utility maximization, and the implied labor supply function is estimated using OLS on linearized data, then there may be a negative small sample bias of several hundred percent for the wage rate coefficient. This study also shows that there may be a negative small sample bias of several hundred percent for instrumental variables estimators using instruments of the socio-demographic type like educational level of the person and his/her parents, geographic location etc. According to the simulation results it seems as if a sample size of something like 10,000 to 20,000 is needed for instruments of this type to work. Thus, many earlier studies using linearization techniques have used estimation methods potentially plagued by a severe negative small sample bias for the wage rate coefficient.

What about the results in MaCurdy et al. (1990)? Do their results not refute the utility maximization hypothesis? A close inspection of their paper shows that they use a rule of thumb to calculate individuals' tax deductions. That is, this variable is measured with error. Simulations in Blomquist (1992) show that this type of measurement error can lead to a severe negative bias for the wage rate coefficient if the Hausman method is used. Thus, one possible reason for their 'negative Slutsky effect' is that they use poor data.

6. Summary

For both methods commonly used to estimate labor supply functions when data are generated by nonlinear budget constraints it is true that the parameter estimates can only be interpreted if they are consistent with utility maximization. However, it is also true for both methods that the relevant objective function can be optimized without imposing constraints guaranteeing parameter estimates that are consistent with utility maximization. Thus, there are no particular constraints inherent in the Hausman method that can explain the difference in results obtained by the Hausman method and the linearization method. Measurement errors in variables, which affect both the Hausman method and the linearization method, and a severe small sample bias for the commonly used instrumental variables technique are likely explanations for the wide divergence in reported results.

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