Suggested Answer of PS#4 in 2014

1. (a)Dividing the both side, we get

$$\begin{aligned} Y/L &= K^{0.5}L^{-05} \\ y &= k^{0.5} \end{aligned}$$

where $y = Y/L$ and $k = K/L$. Note that the population growth rate is equal to zero. Therefore, the steady state condition is

$$sy = \delta k$$

This implies that

$$\begin{aligned} 0.5y &= 0.1k \\ 0.5k^{0.5} &= 0.1k \\ 5k^{0.5} &= k \end{aligned}$$

power by 2 both side of equation.

$$\begin{aligned} 25k &= k^2 \\ k^2 - 25k &= 0 \\ k(k-25) &= 0 \end{aligned}$$

Thus, $k = 25$ is the steady state level of capital stock per capita.

(b)Note that populaton growth rate is equal to zero. Thus, Golden rule level of capital stock per capita can be caluculating by setting MPK=δ. MPK is equal to $0.5k^{-0.5}$. Thus, MPK= δ implies that

$$\begin{aligned} 0.5k^{-0.5} &= 0.1 \\ \frac{0.5}{k^{0.5}} &= 0.1 \\ 0.5 &= 0.1k^{0.5} \\ 5 &= k^{0.5} \\ 25 &= k \end{aligned}$$

Thus, in this case, the golden rule level of capital stock per capita is 25. But keep in mind that steady state capital stock per capita is not always equal to golden rule level of capital stock per capita. This question is just coincidence!

5.a) $Y = K^{\alpha}L^{1-\alpha}$. Taking a derivative with respect to K, we get MPK.
MPK=$\alpha K^{\alpha-1}L^{1-\alpha}$
MPL=$(1-\alpha)K^{\alpha}L^{-\alpha}$

b) At the equilibrium $w = MPL$. The labor share is

$$\frac{wL}{Y} = \frac{(1-\alpha)K^\alpha L^{-\alpha} * L}{K^\alpha L^{1-\alpha}} = \frac{(1-\alpha)K^\alpha L^{1-\alpha}}{K^\alpha L^{1-\alpha}} = 1-\alpha$$

Capital share is

$$\frac{rK}{Y} = \frac{\alpha K^{\alpha-1} L^{1-\alpha} * K}{K^\alpha L^{1-\alpha}} = \frac{\alpha K^\alpha L^{1-\alpha}}{K^\alpha L^{1-\alpha}} = \alpha$$

c)In this case, MPK=$\alpha K^{\alpha-1} L^\beta$ and MPL=$\beta K^\alpha L^{\beta-1}$. Thus,

$$\begin{aligned} \frac{wL}{wL+rK} &= \frac{\beta K^\alpha L^{\beta-1} * L}{\beta K^\alpha L^{\beta-1} * L + \alpha K^{\alpha-1} L^\beta * K} \\ &= \frac{\beta K^\alpha L^\beta}{\beta K^\alpha L^\beta + \alpha K^\alpha L^\beta} \\ & \quad \frac{\beta}{\beta+\alpha} \end{aligned}$$

Similarly,

$$\frac{rk}{rk+wL} = \frac{\alpha}{\alpha+\beta}$$