

Macroeconomics 2014

Problem set #4

You do not have to submit the answer of this problem set. The answer is also available on the class web. This PS is intended to give you some practice for the exam.

1. Consider the following Solow model.

$$Y_t = K_t^{0.5} L_t^{0.5}$$

where K_t is the capital stock and L_t is the population at time t .

saving rate is 0.5

depreciation rate is 0.1

population growth rate is zero

population is always equal to one

(a) Calculating by hand, find the steady state capital stock per capita. (You need to show your process)

(b) Let y be the income per capita. Let i be the investment per capita. The capital stock per capita that maximizes $y - i$ at the steady state is called Golden rule level of capital stock per capita. (Note that $y - i$ is equal to the consumption per capita.) In other words, the golden rule level of capital stock per capita is the capital stock per capita that maximizes the consumption per capita. Find the golden rule capital stock per capita.(Hint read the text book or draw the graph of y and i .)

2. Read page 57-58 of the text. Then answer the following question.

Production function $Y = K^\alpha L^{1-\alpha}$ is called the Cobb-Douglas production function. It was developed by mathematician Cobb and economist Douglas. Note that in the labor market equilibrium, $w = MPL$. In the capital market equilibrium $r = MPK$.

a) Calculate MPL and MPK.

b) capital income share in the GDP accounting is $\frac{rK}{Y}$. Labor income share is $\frac{wL}{Y}$. Show that when the economy's production function exhibits a Cobb-Douglas form, capital income share is α and labor income share is $1 - \alpha$.

c) Consider a more general Cobb-Douglas production function where $Y = K^\alpha L^\beta$. Show that capital income share is $\frac{\alpha}{\alpha+\beta}$ and labor income share is $\frac{\beta}{\alpha+\beta}$.