International Trade PS#3
Suggested Answer

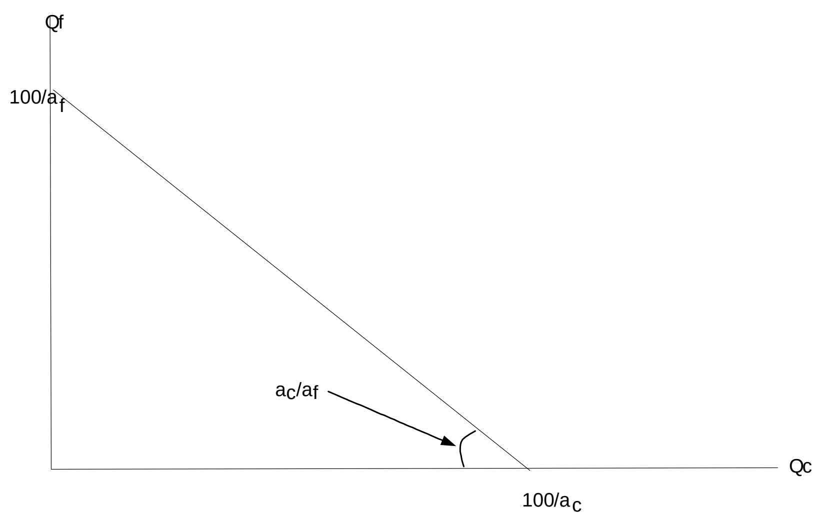
1(a). MRS is equal to $\frac{\text{the marginal utility of } X_c}{\text{marginal utility of } X_f}$.Thus, $\frac{x_f}{x_c}$. At the optimum, this should be equal to p_c/p_f. Thus, we have

$$p_c/p_f = \frac{x_f}{x_c}$$

Solving for x_c/x_f, we have

$$\frac{x_c}{x_f} = \frac{1}{\frac{p_c}{p_f}}$$

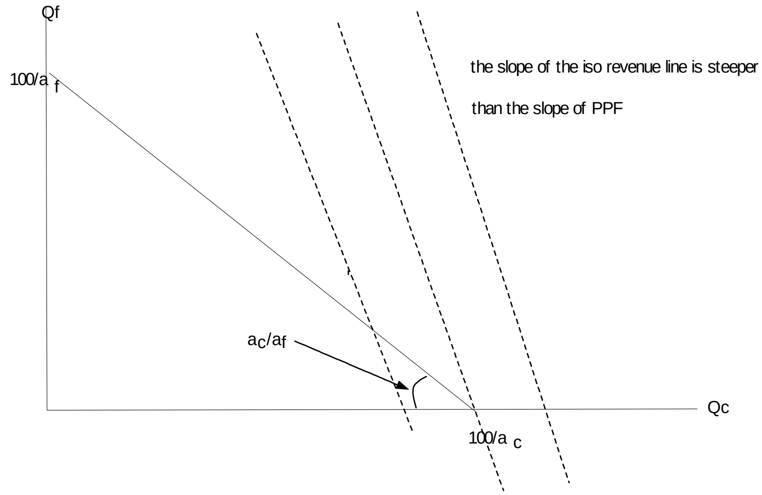
(b)PPF is $L = a_c Q_c + a_f Q_f$. If we measure Q_c on the horizontal axis and Q_f on the vertical axis, then the PPF becomes as follows.



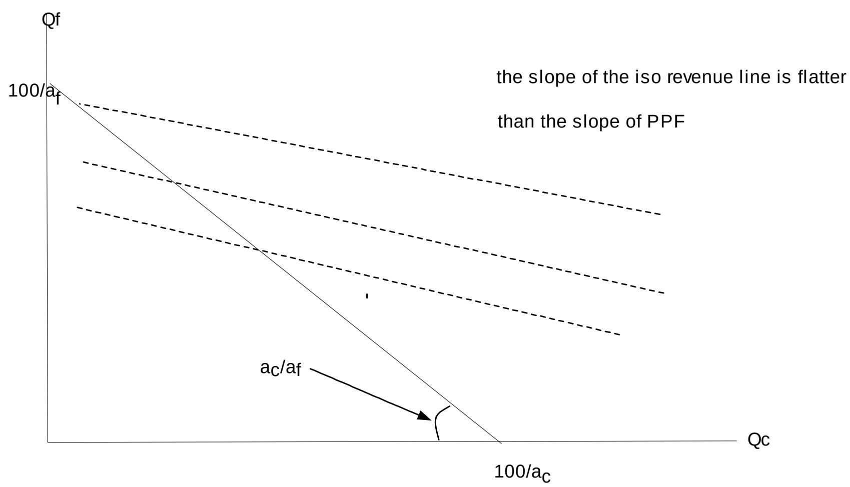
For the optimal production pattern, first write the iso-revenue line. The iso-revenue line is equal to $R = p_c Q_c + p_f Q_f$. Solving for Q_f,the iso-revenue line can be written as

$$Q_f = -\frac{p_c}{p_f} Q_c + \frac{R}{p_f}$$

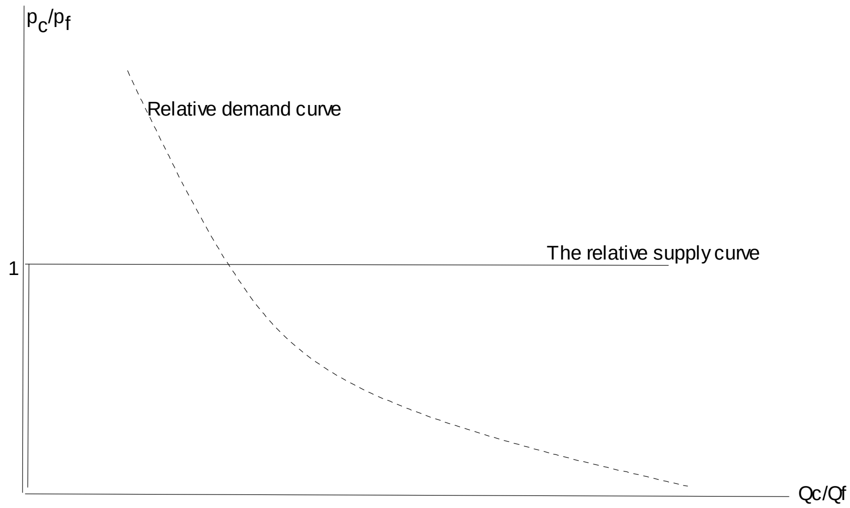
Thus, the absolute value of the slope of the iso-revenue line is p_c/p_f. For the revenue maximizing production pattern, we can decompose into three cases where p_c/p_f is equal to a_c/a_f; p_c/p_f is greater than a_c/a_f; p_c/p_f is lower than a_c/a_f. In the first, case, any point on the PPF is fine. In the second case, it is best to specialize in producing clothing.(See the graph below)



In the third case, it is best to specialize in producing food. (See the graph below)



Now consider how the relative supply, Q_c/Q_f changes as p_c/p_f changes. Since a_c/a_f is equal to one, the relative supply curve is flat at the point where $p_c/p_f = 1$. Thus, the relative supply curve and the relative demand curve become as follows:



(c) From the graph of the relative supply curve and the relative demand curve, the equilibrium p_c/p_f in a closed economy is equal to one.

(d) Note that the budget constraint of the consumers in this country is

$$p_c x_c + p_f x_f = GDP$$

Note that GDP is equal to $p_c Q_c + p_f Q_f$. Thus, the budget constraint becomes

$$p_c x_c + p_f x_f = p_c Q_c + p_f Q_f$$

Dividing by p_f on both side, we have

$$\frac{p_c}{p_f} x_c + x_f = \frac{p_c}{p_f} Q_c + Q_f$$

At the equilibrium, we have $p_c/p_f = \frac{a_c}{a_f}$. Thus, we have

$$\frac{a_c}{a_f} x_c + x_f = \frac{a_c}{a_f} Q_c + Q_f$$

Let L_c be the unit of labor used in clothing sector. Let L_f be the unit of labor used in food sector. Then, $L_c = a_c Q_c$ and $L_f = a_f Q_f$. This implies that $Q_c = \frac{L_c}{a_c}$ and $Q_f = \frac{L_f}{a_f}$. Substituting those question into $\frac{a_c}{a_f} x_c + x_f = \frac{a_c}{a_f} Q_c + Q_f$, we have

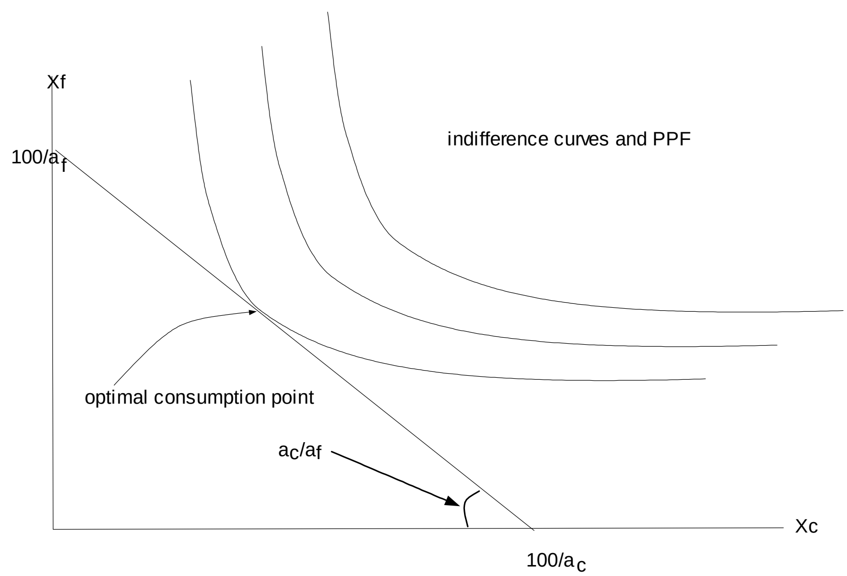
$$\begin{aligned}
\frac{a_c}{a_f} x_c + x_f &= \frac{L_c}{a_f} + \frac{L_f}{a_f}, \\
\frac{a_c}{a_f} x_c + x_f &= \frac{1}{a_f}\{L_c + L_f\} \\
\frac{a_c}{a_f} x_c + x_f &= \frac{1}{a_f} L
\end{aligned}$$

If we draw the graph of $\frac{a_c}{a_f}x_c + x_f = \frac{1}{a_f}L$, this graph becomes identical to the graph of PPF. Thus, x_c and x_f must be on the PPF. Therefore, in a closed economy, we have the following two conditions

(i) x_c and $x_f$is on the PPF

(ii) The MRS is equal to p_c/p_f and the equilibrium p_c/p_f is equal to one. In other words, the slope of the indifference curve at the equilibrium is equal to one.

Those two conditions imply that x_c and x_f must be characterized by the following graph:



In equations, the equilibrium x_c and x_f can be found as follows. First, MRS is equal to p_c/p_f which is one. Thus, $x_c/x_f = 1$.Also, x_c and x_f is on the PPF. Thus,

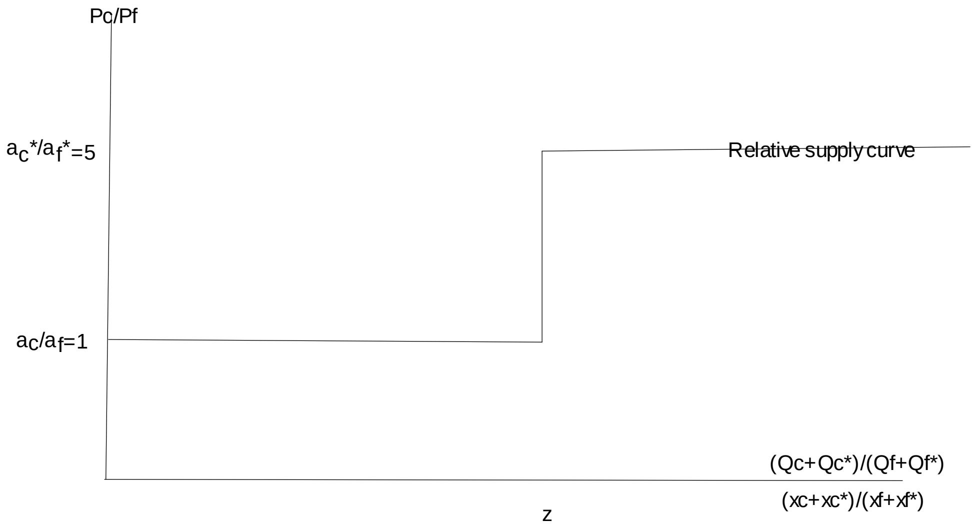
$x_c + x_f = 100.$

From $x_c/x_f = 1$, we have $x_c = x_f$. Substituting this into $x_c + x_f = 100$, we have $x_c = 50$ and $x_f = 50$. In the closed economy, $x_c = Q_c$ and $x_f = Q_f$. Thus, $Q_c = 50$ and $Q_f = 50$.

(e) As in the question (b), we can draw the relative supply curve. The relative supply curve is flat at $p_c/p_f = a_c^*/a_f^*$. Thus, the relative supply curve is flat at 5. This implies that the relative supply curve and the relative demand curve intersect at $p_c/p_f = 5$.Thus, the equilibrium p_c/p_f is equal to 5.

(f) Note that $\frac{a_c}{a_f} < \frac{a_c^*}{a_f^*}$ .Thus, home has a comparative advantage in producing clothing. This implies that foreign has a comparative advantage in producing in food.

(g)The graph of the world relative supply curve becomes as follow:



When p_c/p_f is strictly greater than 1 and less than 5, home will specialize in producing clothing. On the other hand, for foreign, foreign will specialize in producing food. Thus, $Q_c = 100, Q_f = 0, Q_c* = 0$ and $Q_f^* = 400/2 = 200$. This implies that z is equal to

$$z = \frac{Q_c + Q_c^*}{Q_f + Q_f^*} = \frac{100}{200} = \frac{1}{2}$$

As for the world relative demand curve, the world relative demand is defined as

$$\begin{aligned} & \frac{x_c + x_c^*}{x_f + x_f^*} \\ = \quad & \frac{x_c}{x_f + x_f^*} + \frac{x_c^*}{x_f + x_f^*} \end{aligned}$$

For the first term, we divide by x_f and multiply x_f .For the second term, we divide it by x_f^* and multiply by x_f^*. Thus, we have

$$\frac{x_f}{x_f + x_f^*} \frac{x_c}{x_f} + \frac{x_f^*}{x_f + x_f^*} \frac{x_c^*}{x_f^*}$$

Note that $\frac{x_c}{x_f}$ is the relative demand in home country and $\frac{x_c^*}{x_f^*}$ is the relative demand in foreign country. The question assumes that the people in home and people in foreign have the same utility functions. When two countries are engaged in international trade, people in two countries face the same relative price. Since the utility functions are the same in two countries, the relative demand in home and foreign is equal to

$$\frac{x_c}{x_f} = \frac{x_c^*}{x_f^*} = \frac{1}{(p_c/p_f)}$$

Thus, the world relative demand curve is

$$\frac{x_f}{x_f + x_f^*}\frac{1}{(p_c/p_f)} + \frac{x_f^*}{x_f + x_f^*}\frac{1}{(p_c/p_f)}$$

Note that $\frac{x_f}{x_f+x_f^*}$ and $\frac{x_f^*}{x_f+x_f^*}$ can be interpreted as the weight because the sum of those two are equal to one since

$$\frac{x_f}{x_f + x_f^*} + \frac{x_f^*}{x_f + x_f^*} = \frac{x_f + x_f^*}{x_f + x_f^*}$$

. Thus, the world relative demand curve is the weighted average of $\frac{1}{(p_c/p_f)}$. Thus, the world relative demand curve is equal to

$$\frac{1}{(p_c/p_f)}$$

This implies that the world relative demand curve is the same as the relative demand curve in home and the relative demand curve in foreign. This is obvious since the utility functions of people in two countries are the same and they face the same relative price when two countries are engaged in international trade.

Now we need to find the equilibrium relative price. As I explained in the class, there are three possible cases. $p_c/p_f = 1, p_c/p_f = 5$ or p_c/p_f is greater than 1 and less than 5. To check which case applies, first consider the case where $p_c/p_f = 1$. In this case, the world relative demand is also equal to one since the world relative demand curve is $\frac{1}{(p_c/p_f)}$. However, from the graph of the world relative supply curve we know that when p_c/p_f is equal to one, the world relative supply curve cannot be one. When $p_c/p_f = 1$, the world relative supply is lower than $\frac{1}{2}$. This is because z=$\frac{1}{2}$ on the graph. Thus, $\frac{p_c}{p_f} = 1$ is not possible as the equilibrium. How about the case where p_c/p_f is equal to 5. In this case, the world relative demand is $\frac{1}{5.}$. On the other hand, from the graph, the world relative supply cannot be $\frac{1}{5}$. The world relative supply is greater than $\frac{1}{2.}$ when the world relative price is equal to 5. This is because $z = \frac{1}{2}$. This implies that p_c/p_f must be greater than 1 and less than 5. Then, what is the equilibrium relative price exactly? To calculate, note that $z = \frac{1}{2}$ on the graph. Thus, substitute this number in the relative demand curve. We need to find the equilibrium relative price that will generate the equilibruim relative demand that is equal to $\frac{1}{2.}$

$$\frac{1}{2} = \frac{1}{(p_c/p_f)}$$

Solving this equation, we have

$$\frac{p_c}{p_f} = 2$$

Thus, the equilibrium relative price is equal to 2. In fact when p_c/p_f is equal to 2, home will specialize in producing clothing and foreign will specialize in producing food. As the result, the world relative supply is equal to $\frac{1}{2}$. On the other hand, when p_c/p_f is equal to 2, the world relative demand becomes equal to $\frac{1}{2}$. The world economy is at the equilibrium.

(h) The value of the import should be equal to the value of export. Thus, we have

$p_c(Q_c - x_c) = p_f(x_f - Q_f)$. This implies that we have

$$p_c x_c + p_f x_f = p_c Q_c + p_f Q_f$$

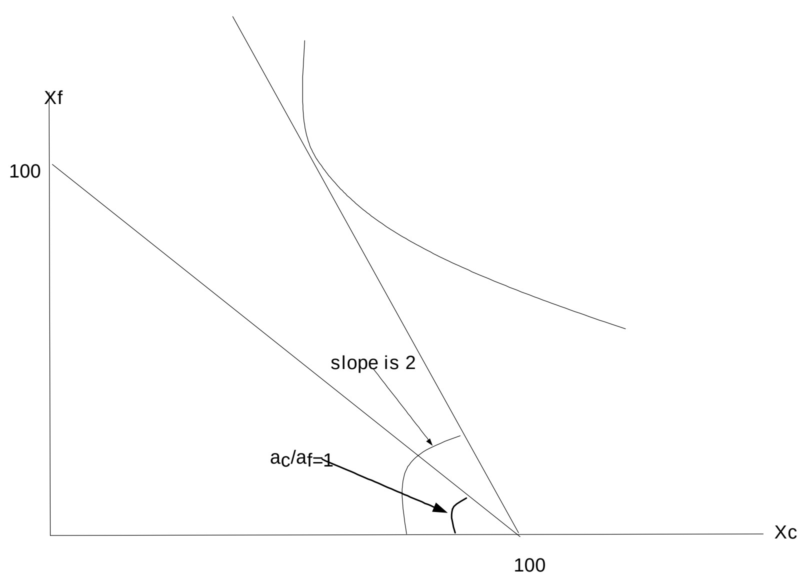
This equation has a direct interpretation. The LHS is the value of the expenditure of clothing and food. The RHS is the value of good produced, which is equal to GDP. Thus, the above equation shows that the value of expenditure should be equal to GDP. By dividing the above equation by p_f, we have

$$\frac{p_c}{p_f} x_c + x_f = \frac{p_c}{p_f} Q_c + Q_f$$

Note that at the equilibrium, $p_c/p_f = 2$ and $Q_c = 100$ and $Q_f = 0$. Thus,

$$2x_c + x_f = 2 \times 100 \tag{1}$$

Graphically, this can be illustrated as follows:



To calculate how much people in home consume clothing and food, notice that x_c and x_f must satisfy the two condition as illustrated by the above graph.

(i) MRS should be equal to 2.

(ii) x_c and x_f should be on the budget constraint.

The condition (i) implies that

$$\frac{x_c}{x_f} = \frac{1}{2}$$

The condition (ii) implies that $2x_c + x_f = 2 \times 100$ from the equation (1). From those two equations, we have

$$\begin{aligned} x_f &= 2x_c \\ 2x_c + x_f &= 200 \end{aligned}$$

Thus, we have, $x_c = 50$ and $x_f = 100$.

Note that the utility function of home is $\log x_c + \log x_f$. When home is a closed economy, we found that $x_c = 50$ and $x_f = 50$. Thus, using excel, we can find that log (50)=1.69897. Thus, log(50)+log(50)=3.39794. On the other hand, after engaged in international trade, $x_c = 50$ and $x_f = 100$. Note that log(100)=2.Thus, log (50)+log(100)=3.69897. 3.6987 is greater than 3.39794.

(i)Note that $Q_c = 100$ and $Q_f = 0$. Thus, home will export clothing 50 units and import food 100 units.

(j) The value of import is equal to the value of export if the following equation holds

$$p_c \times \text{units of exported clothing} = p_f \times \text{units of imported food}$$

This is equivalent to

$$\frac{p_c}{p_f} \times \text{ units of exported clothing} = \text{units of imported food}$$

Since $p_c/p_f = 2$, the LHS is equal to $2{\times}50 = 100$. RHS is 100. Thus, the above equation holds. This implies that the value of export is equal to the value of import.

(k)Using the same argument in question (h), we have the following equation

$$p_c x_c^* + p_f x_f^* = p_c Q_c^* + p_f Q_f^*$$

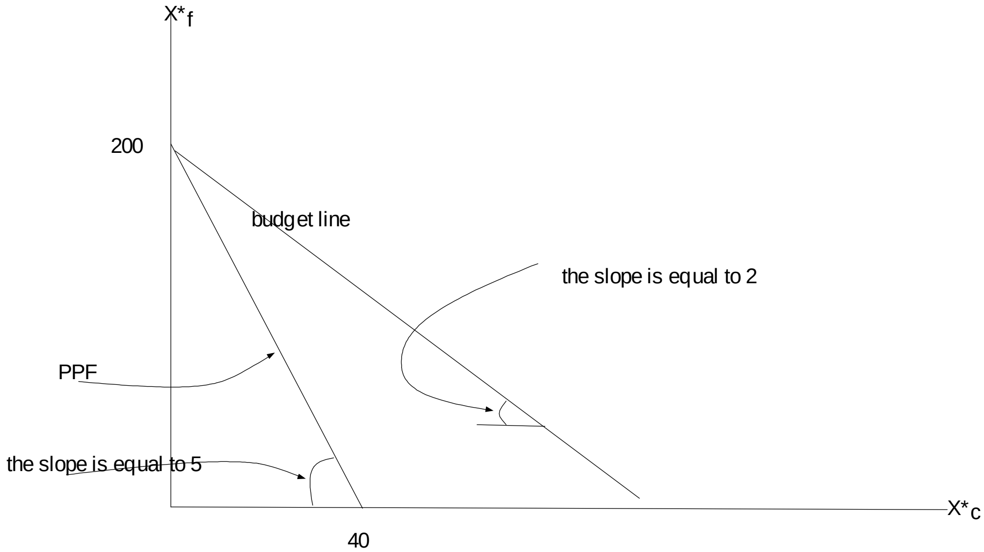
This is equivalent to

$$\frac{p_c}{p_f} x_c^* + x_f = \frac{p_c}{p_f} Q_c^* + Q_f^*$$

Note that $Q_c^* = 0$ and $Q_f^* = 200$. Also notice that $p_c/p_f = 2$.Thus, the budget constraint becomes

$$2x_c^* + x_f^* = 200$$

The graph of the budget line of the foreign becomes as follows:



x_c and x_f are characterized by two conditions.
i) MRS should be equal to p_c/p_f which is equal to 2.
ii) x_c and x_f should be on the budget line.

Translate those two condition into equations, we have the following equations

$$\frac{x_c^*}{x_f^*} = \frac{1}{2}$$
$$2x_c^* + x_f^* = 200$$

By solving those two equations, we have $2x_c^* = x_f^*$ and $2x_c^* + x_f^* = 200$. Thus, we have $x_c^* = 50$ and $x_f^* = 100$.

(l) Since $Q_c^* = 0$ and $Q_f^* = 200$, the amount of imported clothing is 50 and the amount of exported food is $200 - 100 = 100$. Thus, foreign import 50 units of clothing and export 100 units of food

(m) Note that home will export clothing 50 units and import food 100 units. Thus, the amount of import of foreign is equal to export of home. The amount of export of foreign is equal to the amount of export of home.