Specific Factor Model

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Motivation

- Ricardian Model assumes that there is only one type of factor for production
- No issue on Income Redistribution
- No-non-tradable sector
- Both are important for considering the implication for trade policy
- Specific Factor model allows such consideration

The model overview

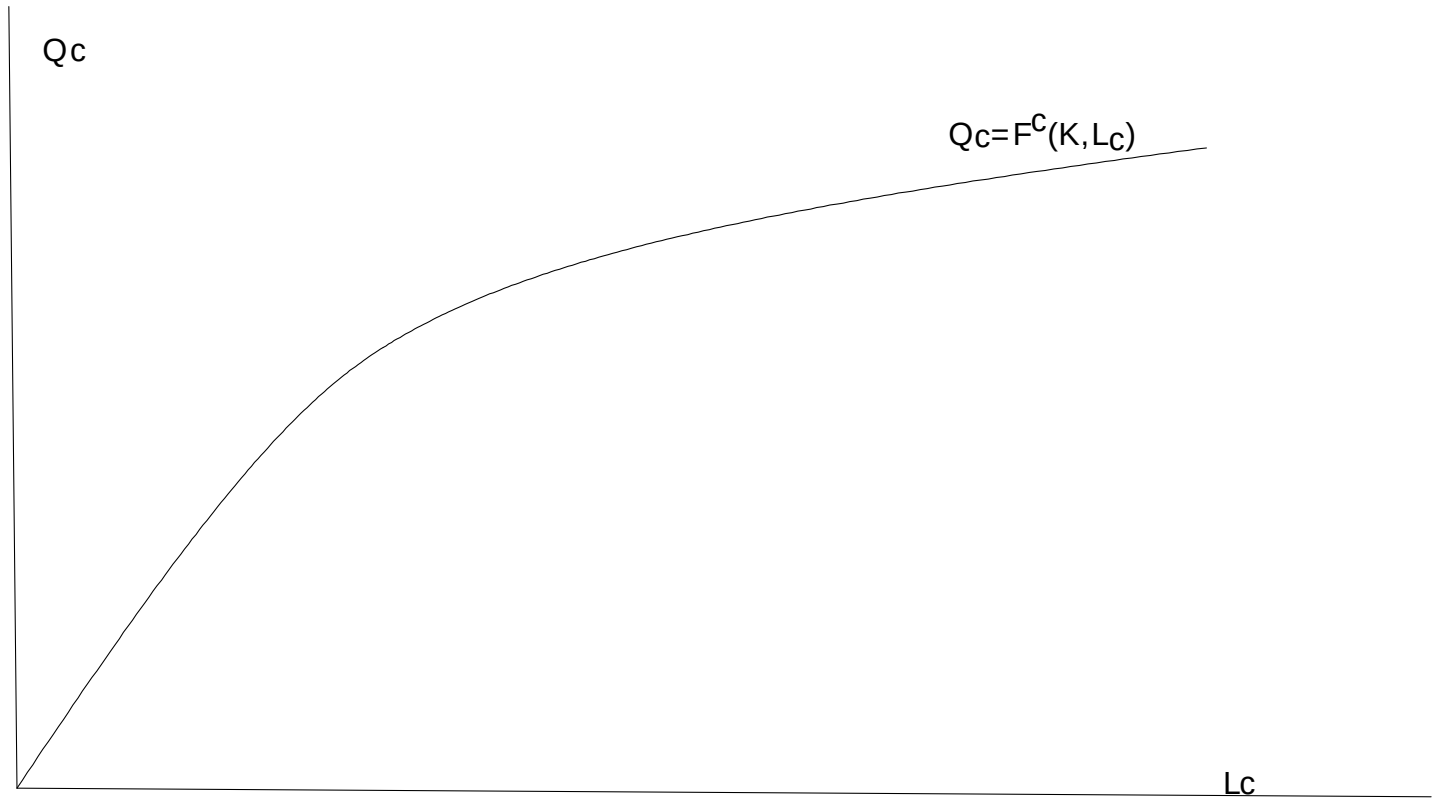
- Two sectors, Two goods, three factor model, two sector model
- There are specific factor for each sector.
- Those factors are immobile between two sector
- Labor is mobile between two sectors
- Competitive market, each industry behave as a price taker

Assumptions

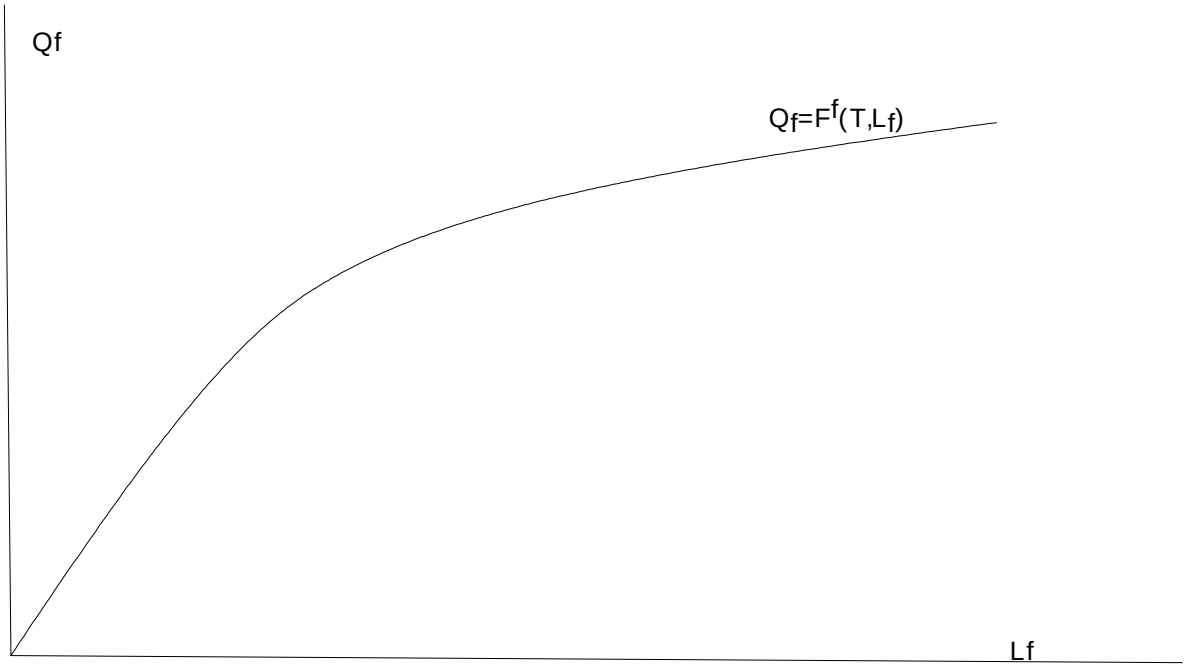
- Production function of each sector is $Q_c = F^c(K, L_c)$ and $Q_f = F^f(T, L_f)$
- F^c is a production function of the clothing sector
- F^f is a production function of the food sector
- Assumption 1:F^c and F^f exhibits constant returns to scale
- Assumption 2: Capital, K, and land, T, are immobile and labor are mobile between two sectors
- Assumption 3:F^c and F^f are increasing function of K, T, L_c and L_f
- Assumption 4: the law of diminishing marginal product holds for K, T, L_c and L_f
- Assumption 5: the total labor in this country is $\overline{L}$
- Assumption 6:$F^c(K, 0) = 0$ and $F^f(T, 0) = 0$. In other words, when no labor is used, the output in that sector is zero

Assumptions(2)

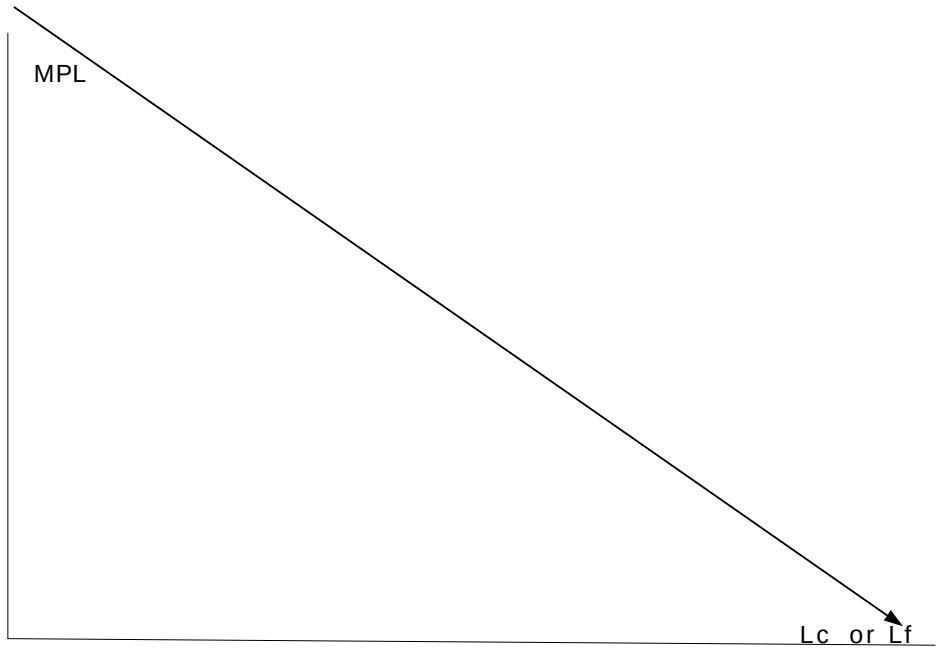
- The assumption 3 and 4 implies that the production function has the following shape



- Summarily for Q_f

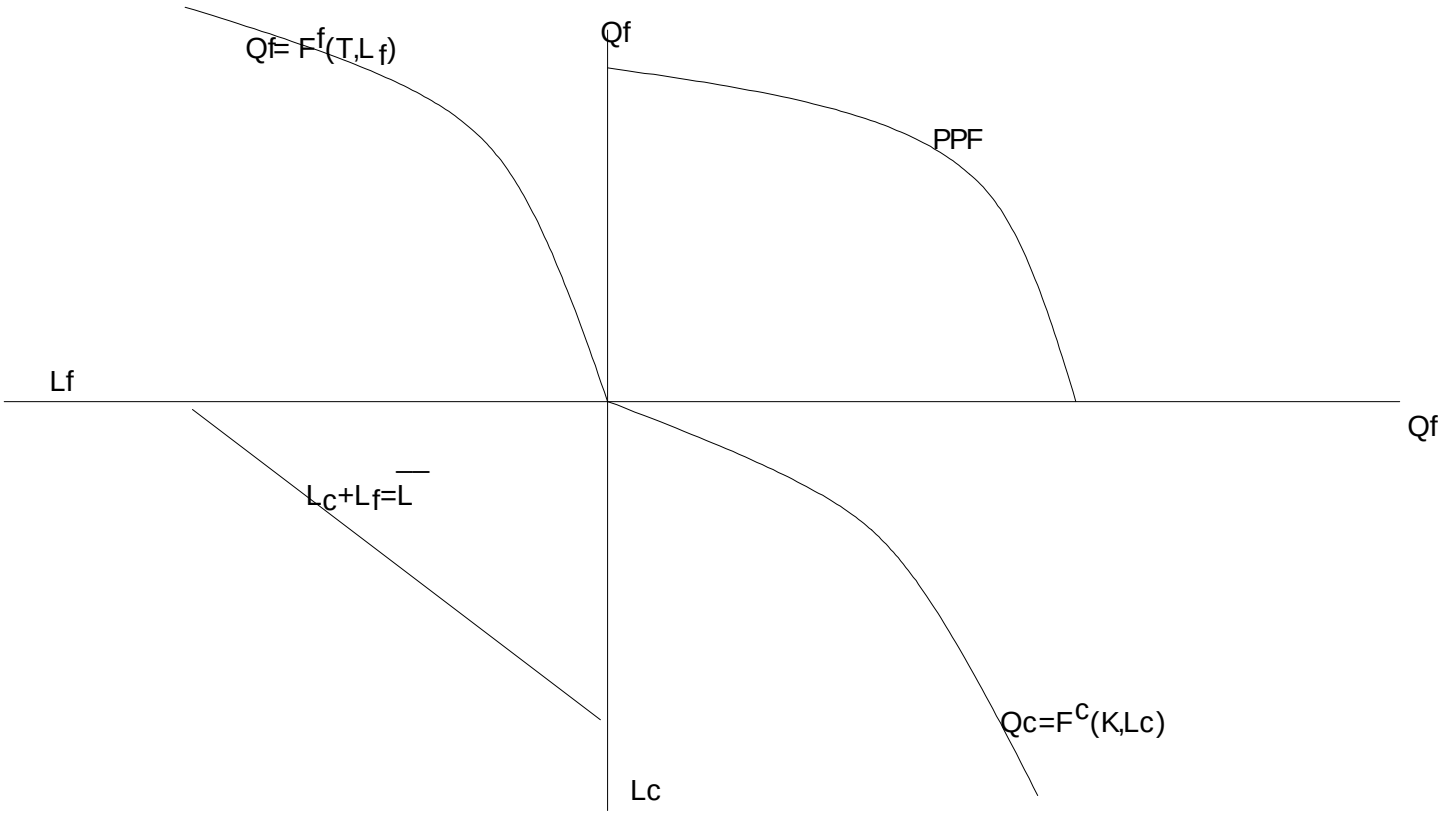


- The marginal product of labor in each sector has the following graph



PPF

- PPF becomes as follows(see the explanation on the blackboard)



Labor demand(1)

- p_c and p_f are determined in the international market.
- Each sector chooses labor to maximize its profit given $p_c$and p_f This implies that

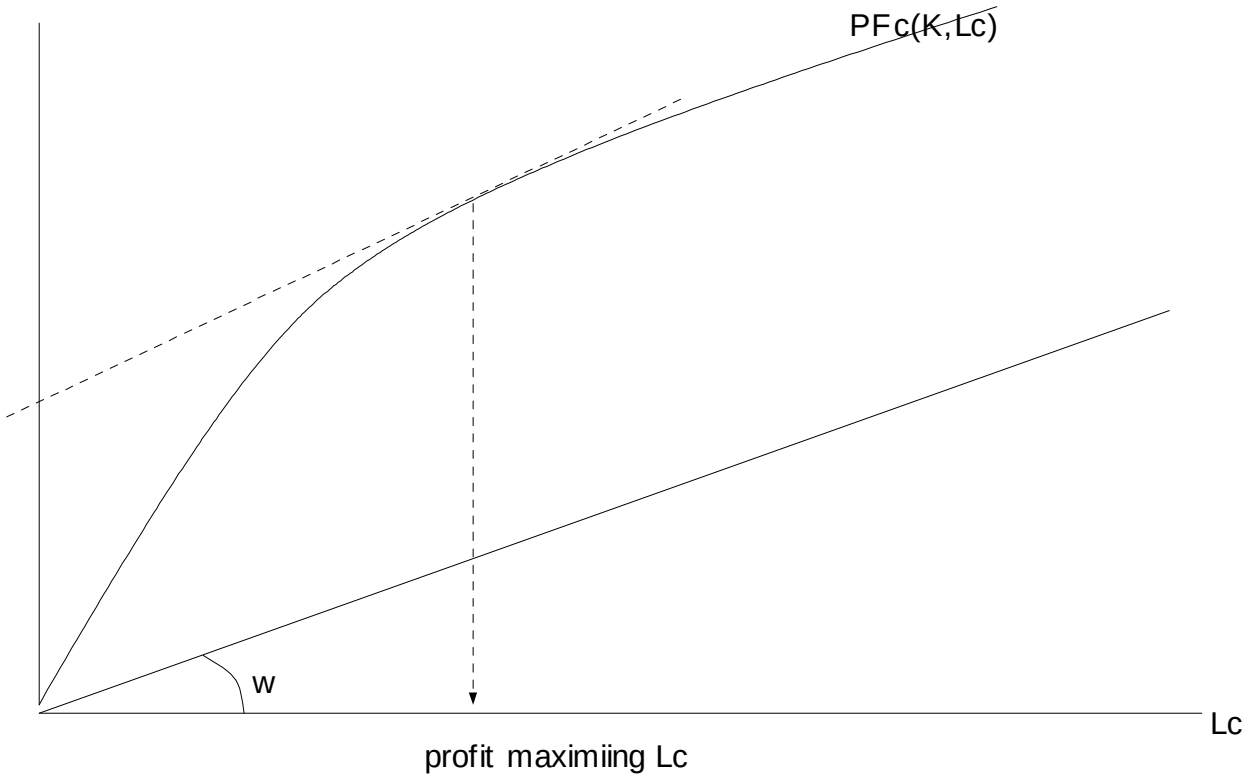
$$\max_{\{L_c\}} p_c F^c(K, L_c) - wL_c$$

- The first order condition is

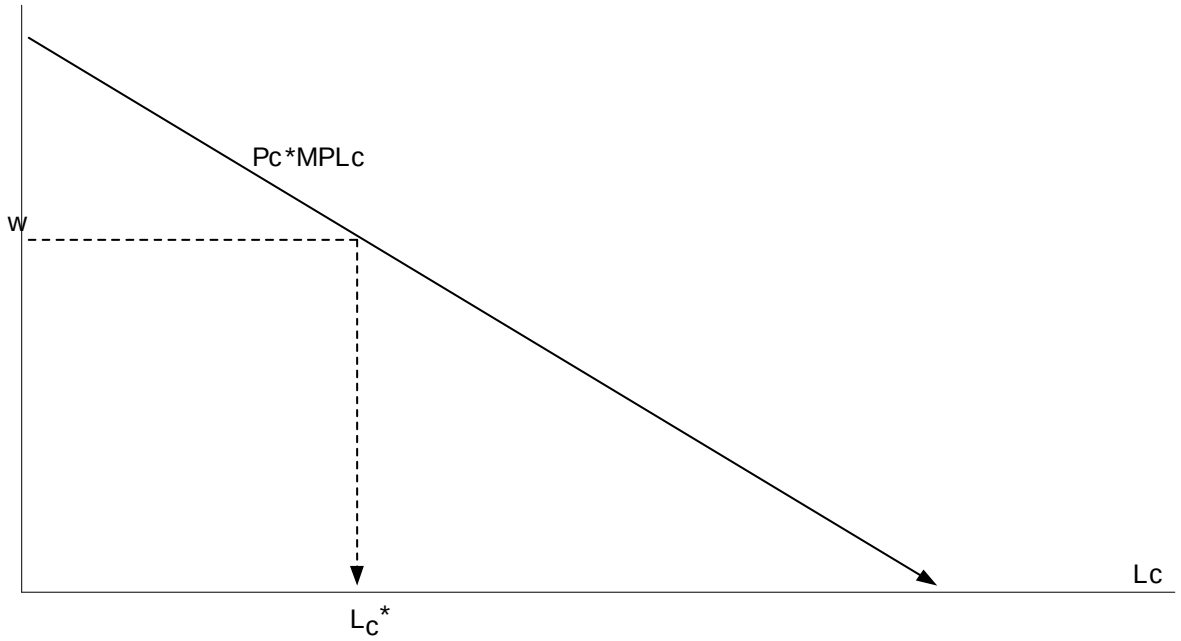
$$p_c \frac{\Delta F^c(K, L_c)}{\Delta L_c} - w = 0$$

- The above equation determines the labor demand in food sector.

- Graphically, this can be explained as follows:



The value of the marginal product of labor and the nominal wage level



Labor Demand (1)

- Similarly, the labor demand in food sector is determined as follow

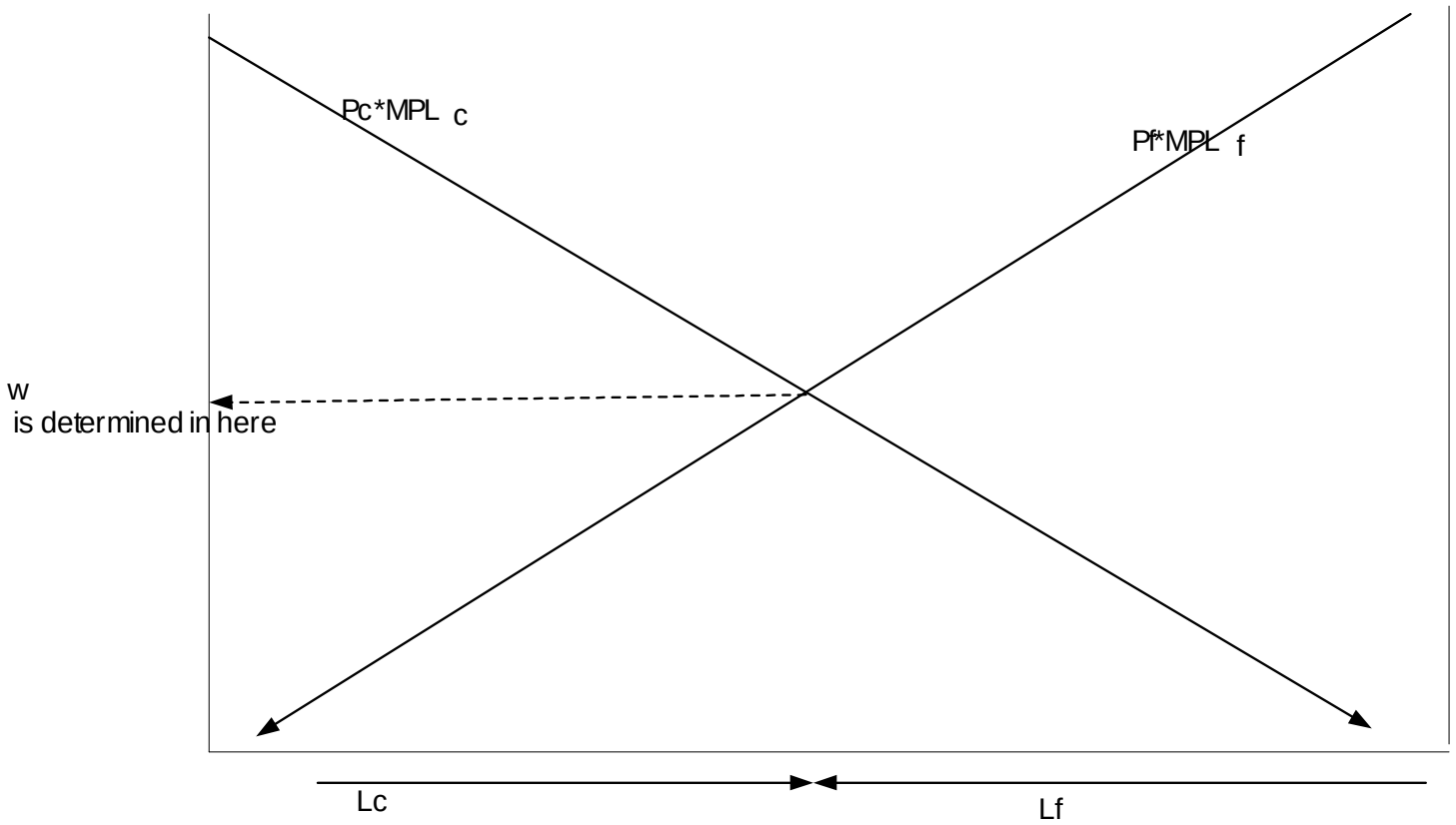
$$p_f \frac{\Delta F^f(K, L_f)}{\Delta L_f} - w = 0$$

- Denote the labor demand function in the clothing sector as $L_c^d(p_c, w)$
- Denote the labor demand function $L_f^d(p_f, w)$

Labor Demand (2)

- Note that p_c and p_f are determined in the international market and they are fixed in here
- If $L_c^d(p_c, w) + L_f^d(p_f, w) > \overline{L}$, it implies that total labor demand is greater than the available amount of the labor in the economy
- $L_c^d(p_c, w) + L_f^d(p_f, w) < \overline{L}$, the total amount of available labor is greater than the total labor demand. Some people are unemployed
- w will respond to equate the labor demand and labor supply
- At the equilibrium, $L_c^d(p_c, w) + L_f^d(p_f, w) = \overline{L}$

Graphically, this is illustrated as follows:



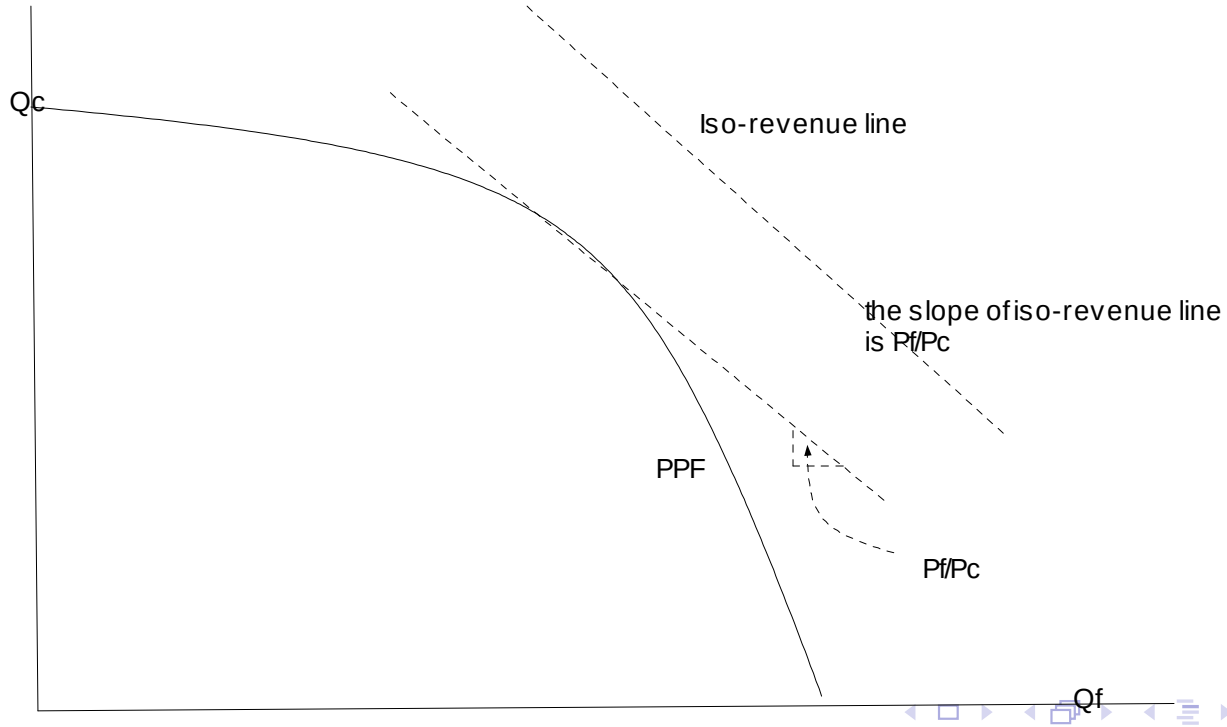
- Once L_c^* and Lf^* are determined, then Q_c and Q_f are determined from

$$Q_c = F^c(K, L_c) \text{ and } Q_f = F^f(T, L_f)$$

- Note that at the equilibrium, $P_c \times MPL_c = p_f \times MPL_f$ why?
- As we discuss later, this has an important implication for PPF.

Another look at PPF

- Suppose that two sectors are owned by one big firm.
- In other words, one big firm has two divisions, clothing producing division and food producing division.
- Assume that this big firm maximizes the total revenue, $p_c Q_c + p_f Q_f$
  p_c, p_f and PPF
- Then the slope of the PPF should be equal to P_f / P_c



- On the other hand, the slope of the PPF is calculated as follows:
- We know that when L_f is increased by one unit, Q_f increases by $\frac{\Delta F^f}{\Delta L_f}$, the marginal product of labor, MPL_f
- Similarly, when L_f is increased by one unit, Q_c increases by $\frac{\Delta Fc}{\Delta Lc}$, MPL_f.

- By the definition, the slope of PPF is the change of Q_c when the Q_f is decreased by one unit.
- In order to decrease Q_f, we need to decrease the amount of labor in food division by $\frac{1}{\frac{\Delta Fc}{\Delta Lc}}$
- This decreased labor can be used in the clothing division.
- Thus, Q_c will increase $\frac{\Delta Fc}{\Delta Lc} \times \frac{1}{\frac{\Delta Fc}{\Delta Lc}} = \frac{MPL_c}{MPL_f}$
- In other words, the slope of PPF is $\frac{MPL_c}{MPL_f}$
- This implies that

$$\frac{MPL_c}{MPL_f} = \frac{P_f}{P_c}$$

- But this implies that

$$P_f \times MPL_f = P_c \times MPL_c$$

- This is what we have in the decentralized case.
- The allocation determined by one big firm case is equivalent to the decentralized case

Comparative Statics (1)

- Now consider what will happen to the economy when p_c and p_f increases 10 percent ?
- In this case, $p_c \times MPL_c$ curve shifts up 10 percent
- $p_f \times MPL_f$ curve shifts up 10 percent
- The nominal wage increases 10 percent
- The equilibrium amount of labor in clothing sector and food sector L_c^* and L_f^* are the same
- Q_c and Q_f are the same
- This also can be seen easily in PPF's graph.
- Since P_f / P_c does not change, the optimal Q_f and Q_c does not change.
- The nominal wage goes up 10 percent but the P_c and P_f also increases by 10 percent.
- The real wage, w / p_c or w / p_f are the same.

Comparative Statics (2)

For capital owner in the clothing sector,
The nominal income of the capital owner is $P_c Q_c - wL_c$
L_c is the same and Q_c is also the same
Thus, the nominal income of capital owner increases by 10 percent.
The real income of capital owner is the same
Similarly, the nominal income of land owner in food sector increases by 10 percent
The real income of the land owner is the same.

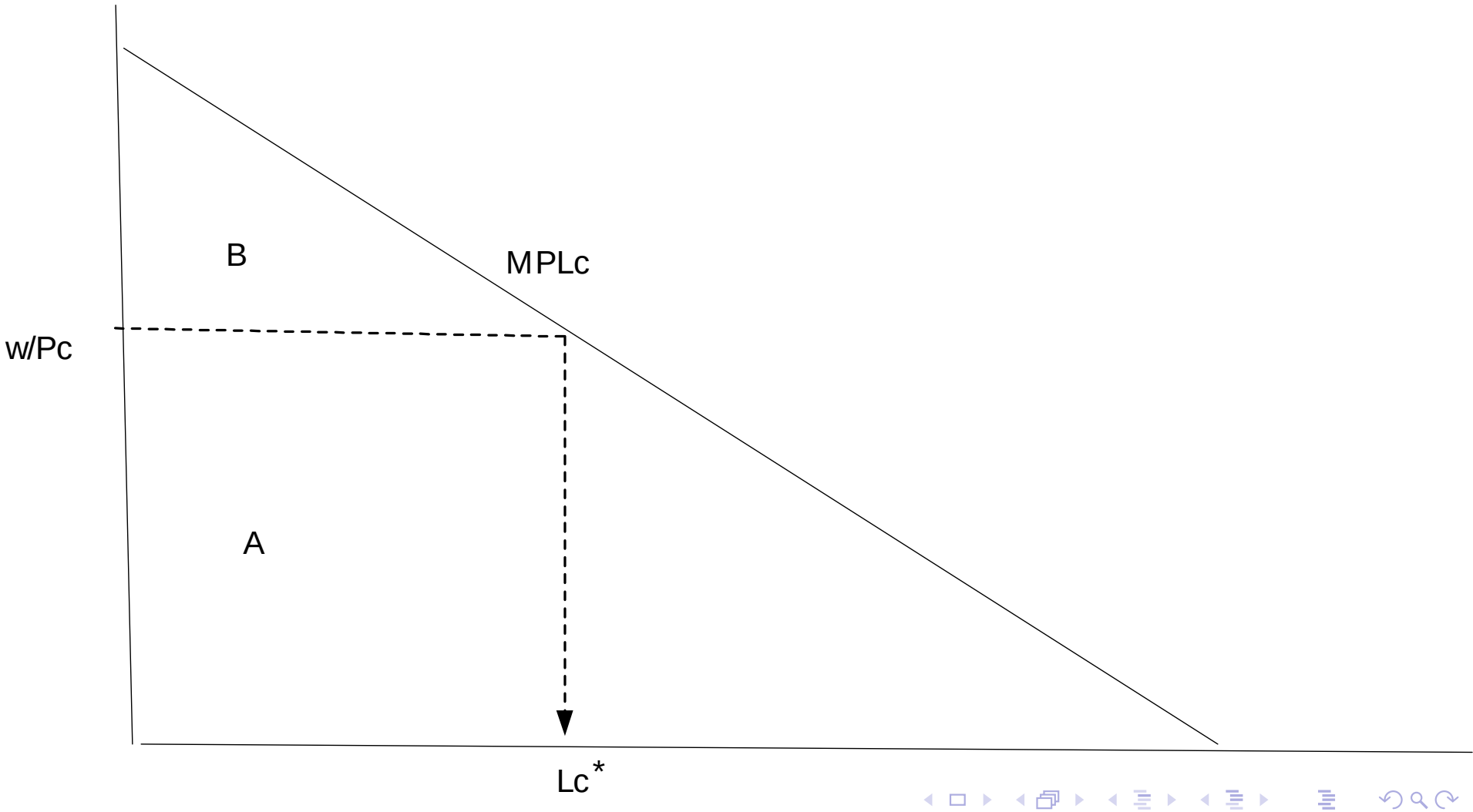
Comparative Statics (3)

Graphically, this can be illustrated as follows.
In the clothing sector, the optimal amount of labor, L_c is determined as follows

$$\begin{aligned} P_c \times MPL_c &= w \\ MPL_c &= w/P_c \end{aligned}$$

Interpretation of the graph

Consider the following graph



- From the fundamental theorem of Integration, the area A+B is equal to

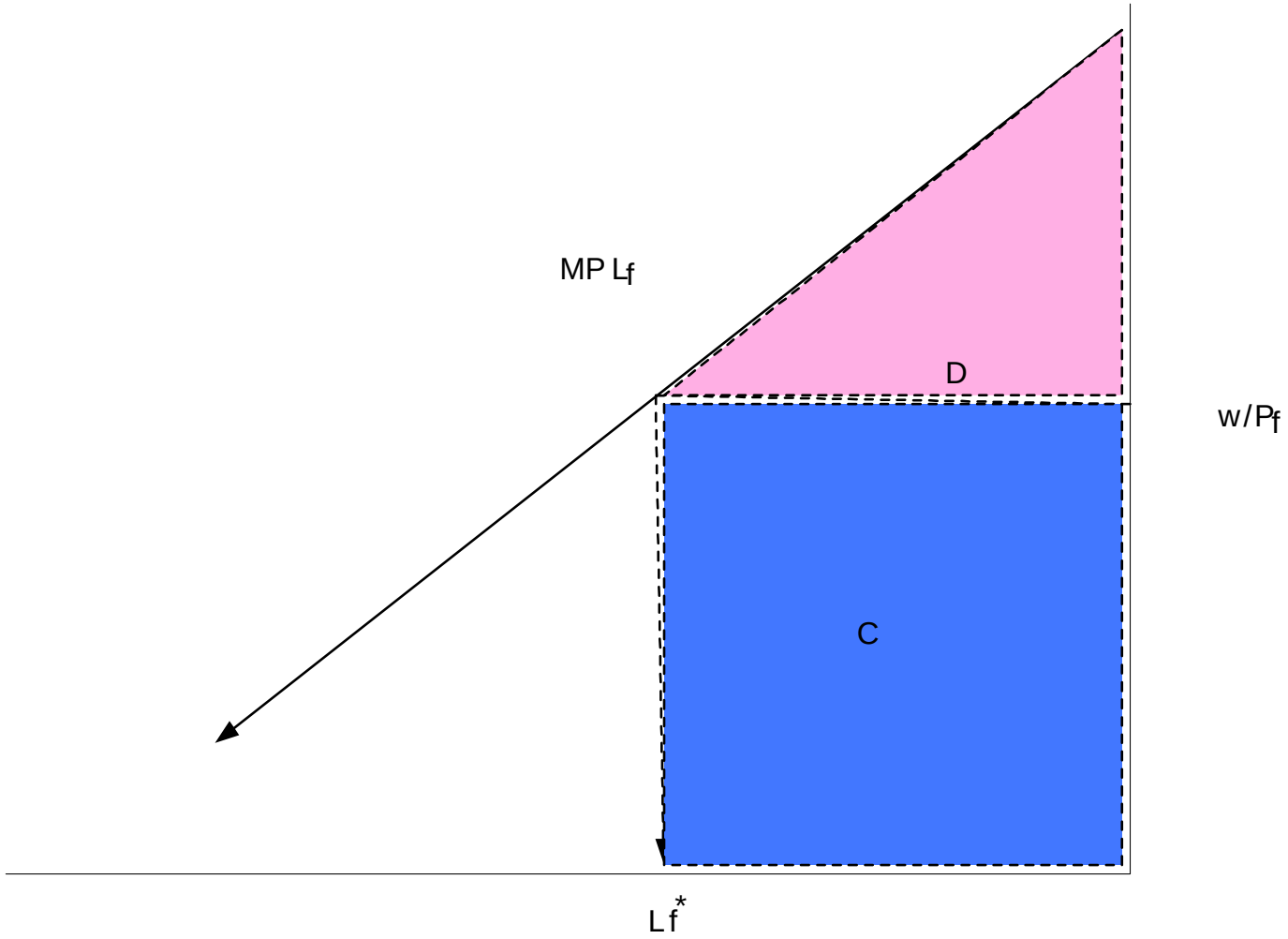
$$\begin{aligned} & \int_0^{L_c^*} [MPL_c] dLc \\ = & \int_0^{L_c^*} [MPL_c] dLc \\ = & \int_0^{L_c^*} [\frac{\Delta F^c}{\Delta L_c}] dLc \\ = & F^c(K, x)|_0^{L_c^*} \\ = & \{F^c(K, L_c^*) - F^c(K, 0)\} \\ = & F^c(K, L_c^*) \end{aligned}$$

- Thus, the area A+B is equal to the total output in the clothing sector
- The area A is $\frac{w}{P_c} \times L_c^*$, which is equal to what the workers in the clothing sector receives in real term
- The area B is

$$\begin{aligned} & Q_c - \frac{w}{P_c} \times L_c^* \\ = \quad & \{P_c Q_c - w \times L_c^*\} / P_c \end{aligned}$$

- The inside of the bracket is the total sales minus the payment to workers.
- Thus, the inside of the bracket is the income of capital owner in clothing sector.
- The inside of the bracket divided by P_c is the real income of capital owner in food sector measured in terms of units of clothing.
- In other words, the area B is the real income of capital owner measured in the units of clothing.

- Similarly, we can draw the graph of MPL_f and w/P_f



- The area C+D shows the total output in food sector
- The area C is the real income of workers in food sector measured in unit of food
- The area D is the real income of the land owner measured in unit of food
- From the above two graphs, it is clear that the real income of capital owner and land owner does not change when w/p_c and w/p_f do not change.

Comparative Statics (4)

- Now consider what will happen to the economy when only p_c increases.
- In this case, $p_c \times MPL_c$ curve shift up 10 percent
- $p_f \times MPL_f$ curve does not move at all
- The nominal wage increases only less than 10 percent
- w/p_c decreases
- w/p_f increases
- The amount of labor used in the clothing sector, L_c^* will increase
- The amount of labor used in the food sector,L_f^* will decrease.
- The real income of capital owner will increase
- The real income of land owner will decrease.
- The real income of workers is unknown.

Summary

- In the specific factor model, the PPF becomes curved
- The slope of PPF is equal to MPL_c / MPL_f
- At the equilibrium, this is equal to P_f / P_c
- Simultaneous increase of both P_c and P_f will not affect the real income of the capital owner, the land owner, workers.
- When only P_c increases, the real income of the capital owner increases but the real income of the land owner decreases.
- The effect on the real income of workers is unclear.