International Trade
PS#2
Due:October 29th in class

1. Consider the utility maximization problem of a consumer. Assume that her utility function is

$$U = \ln x_c + 2\ln x_f$$

where x_c is the consumption of clothing and x_f is the amount of consumption of food. Assume that her budget constraint is $p_x x_c + p_f x_f = m$.

(a) Explain the concept of the indifference curve. What is the definition of the indifference curve between x_f and x_c.

(b) Explain the concept of the marginal rate of substitution between x_f and x_c. What is the definition of it? What does the marginal rate of substitution try to express?

(c) Measure the x_f on the horizontal axis and x_c on the vertical axis. Then, the marginal rate of substitution between x_c and x_f can be the slope of the indifference curve. Why? Explain.

(d) When x_f is measured on the horizontal axis, the absolute value of the marginal rate of the substitution between x_c and x_f can be defined as

$$\frac{\frac{\Delta U}{\Delta x_f}}{\frac{\Delta U}{\Delta x_c}}$$

where $\frac{\Delta U}{\Delta x_f}$ is the derivative of U with respect to x_f and $\frac{\Delta U}{\Delta x_c}$ is the derivative of U with respect to x_f. Explain why the marginal rate of the substitution can be express by the above equation.

(e) Show that the marginal rate of the substitution becomes

$$\frac{2x_c}{x_f}$$

(f) When the x_f is measured on the horizontal axis and x_c is measured on the vertical axis, the absolute value of the slope of the budget constraint is p_f/p_c. Explain why.

(g) When the consumer is maximizing her utility, the marginal rate of substitution(MRS) becomes equal to the price ratio, p_f/p_c. Also, x_c and x_f must be on her budget constraint. This implies that we have two equation and two unknown variables, x_c and x_f.

$$\begin{aligned} \frac{2x_c}{x_f} &= \frac{p_f}{p_c} \\ p_f x_f + p_c x_c &= m \end{aligned}$$

To solve those two equation, first rewrite the first equation as follows:

$$x_f = \frac{p_c}{p_f} 2x_c$$

Then, plug $x_f = \frac{p_c}{p_f} 2x_c$ into $p_f x_f + p_c x_c = m$. Then solve for x_c. Show that

$$x_c = \frac{1}{3} \frac{m}{p_c}$$

(You need to show the process in the answer).

Show that

$$x_f = \frac{2}{3} \frac{m}{p_f}$$

(You need to show the process)

(h)Calculate x_f/x_c. Show that as the relative price of food over clothing, p_f/p_c, increases, the relative demand x_f/x_c will decrease.

(h) In general, the following utility function is called the Cobb-Douglas utility function

$$U = \alpha \ln x_c + \beta \ln x_f$$

where a and β are some constant number. Show in this case that the marginal rate of the substitution becomes

$$\frac{\frac{\Delta U}{\Delta x_f}}{\frac{\Delta U}{\Delta x_c}} = \frac{\beta}{\alpha} \frac{x_c}{x_f}$$

(i) At the optimum, the MRS should be equal to the price ratio. Also x_c inc. x_f must be on the budget line. This implies that we have two equations

$$\begin{aligned} \frac{\beta}{\alpha} \frac{x_c}{x_f} &= \frac{p_f}{p_c} \\ p_f x_f + p_c x_c &= m \end{aligned}$$

As in the answer in (g), solve for x_c and x_f as a function of $p_f, p_c, m, \alpha, \beta$.(You need to show the process.)

(j)Show that in the above case, the expenditure share of x_c in income, which is $p_c x_c/m$, is

$$\frac{p_c x_c}{m} = \frac{\alpha}{\alpha + \beta}$$

Also show that the expenditure share of x in income, which is $p_f x_f/m$ is

$$\frac{p_f x_f}{m} = \frac{\beta}{\alpha + \beta}$$

(You need to show the process)

(k) First the calculate the relative demand of x_f over $x_c, x_f/x_c$. Show that x_f/x_c decreases as p_f/p_c increases in the general Cobb-Douglas utility function case.

2. To maximize the utility subject to the budget constraint, there is another way to solve it. Consider the following utility maximization problem

$$\begin{aligned} \max U &= \alpha \ln x_c + \beta \ln x_f \\ \text{subject to } p_c x_c + p_f x_f &= m \end{aligned}$$

In the above formula, "subject to" means that there is a constraint for x_c and x_f such that $p_c x_c + p_f x_f = m$.

(a) First to solve this problem, rewrite the budget constraint by solving for x_f

$$x_f = \frac{m - p_c x_c}{p_f}$$

Then substitute this x_f into the utility function. Then we have

$$\max U = \alpha \ln x_c + \beta \ln[\frac{m - p_c x_c}{p_f}]$$

Since the constraint is incorporated into the utility function, what we need to do is to choose x_c which maximize $U = \alpha \ln x_c + \beta \ln[\frac{m-p_c x_c}{p_f}]$. Then, what we need to do is to take a derivative with respect to x_c and set it equal to zero. Note that the derivative of $\alpha \ln x_c$ with respect to x_c is equal to $\frac{\alpha}{x_c}$. Also note that the derivative of $\beta \ln[\frac{m-p_c x_c}{p_f}]$ with respect to x_c is

equal to

$$\frac{\beta}{\frac{m-p_c x_c}{p_f}} \times [-\frac{p_c}{p_f}]$$

Thus, the derivative of U with respect to x_c becomes

$$\frac{\alpha}{x_c} - \frac{p_c}{p_f} \frac{\beta}{\frac{m-p_c x_c}{p_f}}$$

At the optimum, this must be equal to zero. Thus we have,

$$\frac{\alpha}{x_c} - \frac{p_c}{p_f} \frac{\beta}{\frac{m-p_c x_c}{p_f}} = 0$$

Now, solve for x_c in the above equation. Show that your answer coincides with the answer in the question 1 (i).

3. Consider a big firm that has two divisions, food producing division and clothing production division. In this firm, there are L unit of labor. Assume that the manager of this firm can determine how many of L workers work food producing division and how many of L work clothing producing division. Assume that the market price of clothing and food are determined in the market and they are fixed from the point of this firm (price taker assumption).Assume that in order to produce one unit of clothing in this firm, it needs a_c units of labor. Also assume that in order to produce one unit of food in this firm, it

needs a_f units of labor. Let Q_c is the amount of clothing produced in this firm. Let Q_f be the amount of food produced in this firm.

(a) Draw the production possibility frontier by measuring Q_f on the horizontal axis and Q_c on the vertical axis.

(b) Draw the iso-revenue line by measuring Q_f on the horizontal axis and Q_c on the vertical axis.

(c) Show that the absolute value of the slope of iso-revenue line is equal to p_f/p_c.

(d)Show that the absolute value of the slope of the production possibility frontier is a_f/a_c when Q_f is measured on the horizontal axis and Q_c is measured on the vertical axis.

(e) Calculate the revenue maximizing Q_c and Q_f when p_f/p_c is greater than a_f/a_c. Show that in this case, $\mathrm{Q}_c = 0$ and $\mathrm{Q}_f = \frac{L}{a_f}$.

(f)Calculate the revenue maximizing Q_c and Q_f when p_f/p_c is smaller than a_f/a_c. Show that in this case, $\mathrm{Q}_f = 0$ and $\mathrm{Q}_c = \frac{L}{a_c}$.