

Heckscher Ohlin Model

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In the Ricardian model, only the technological difference is the source of international trade

In reality, factor abundance and scarceness seem to be the source of international trade

In other words, international resource allocation seems to be an important source of trade

Heckscher Ohlin model explains such a trade pattern

Assumptions

Two goods, two sectors, two factor model

In two countries, technologies are the same.

Those two countries have the same production functions

Those two countries have the same utility functions.

As a result, the same relative demand curve

Two factors are capital and labor

One sector is capital intensive sector

The other sector is labor intensive sector

Assumptions (2)

Assume that clothing sector is capital intensive sector

Assume that food sector is labor intensive sector

Capital intensive sector uses both capital and labor, but it intensively uses more capital than labor

Labor intensive sector uses both capital and labor, but it intensively uses more labor than capital

Let a_{cK} be the unit of capital needed to produce one unit of clothing

Let a_{cL} be the unit of labor needed to produce one unit of clothing

Let a_{fK} be the unit of capital needed to produce one unit of food

Let a_{fL} be the unit of labor needed to produce one unit of food

Assumptions (3)

There are \bar{K} units of capital and \bar{L} unit of labor in home country
Each industry behaves competitively

We assume that

$$\frac{a_{fK}}{a_{fL}} < \frac{a_{cK}}{a_{cL}}$$

We assume that at the equilibrium, both industries are producing its output. Thus, we assume that capital labor ratio in home country is not extreme

$$\frac{a_{fK}}{a_{fL}} < \frac{\bar{K}}{\bar{L}} < \frac{a_{cK}}{a_{cL}}$$

The equilibrium production and consumption in a small open economy

As the analysis of the Ricardian model and the Specific factor model show, the equilibrium production level can be obtained as the revenue maximizing point given the PPF and iso-revenue line.

From the optimal production point on the PPF, we can find the optimal consumption given the country's budget line.

PPF is defined as combinations of Q_c and Q_f that can be produced in this country given the factor supply of \bar{K} and \bar{L}

When Q_c and Q_f are produced, the amount of labor needed is

$Q_c a_{cL} + Q_f a_{fL}$. This must be lower than L

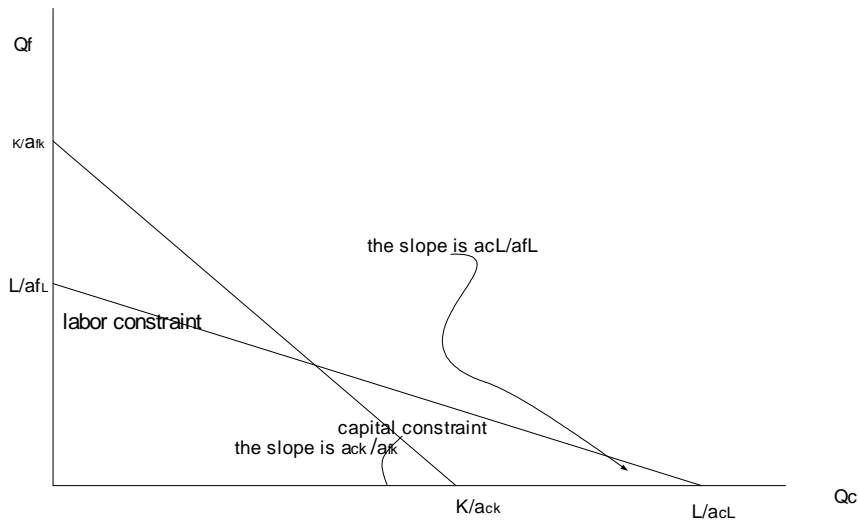
Thus, we have

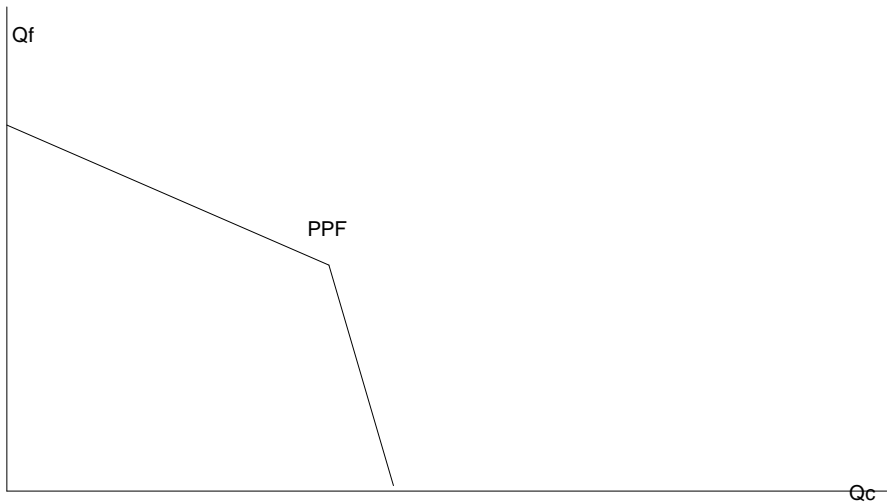
$$Q_c a_{cL} + Q_f a_{fL} \leq \bar{L}$$

Similarly, the amount of capital needed is $Q_c a_{cK} + Q_f a_{fK}$. This must be lower than \bar{K} . Thus, we have

$$Q_c a_{cK} + Q_f a_{fK} \leq \bar{K}.$$

Graphically, this can be explained as follows:



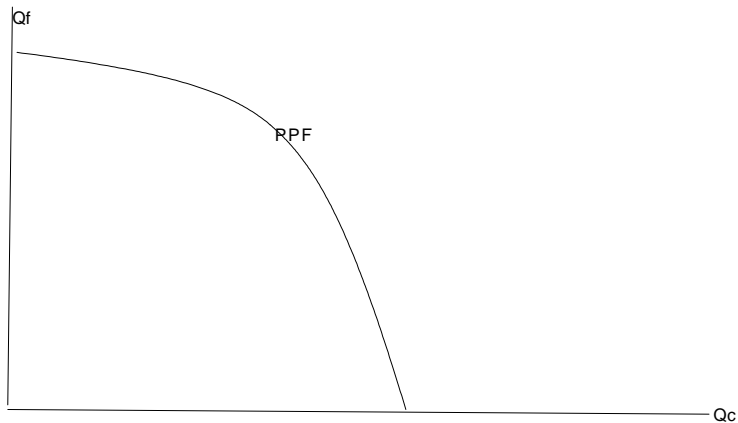


In this model, we have assumed that each sector uses the labor and capital with the fixed ratio (but with different ratios in two sectors).

In other word, we have assumed that it is not possible to substitute one factor with the other in each sector.

Once we assume that each sector can substitute one factor with other factor, PPF becomes more curved as follows:

PPF with factor substitution in each sector



Although assuming factor substitution in each sector is more general than assuming that each sector uses two factor with fixed ratio, it is difficult to solve analytically when factor substitution is allowed.

In fact, in order to show the essence of important theorems in HO theory, the assumption of factor substitution is not needed.

Thus, in this section, we assume that each sector uses two factor with fixed ratio. This implies that PPF has a cone and not smooth like the above graph

Iso-revenue line

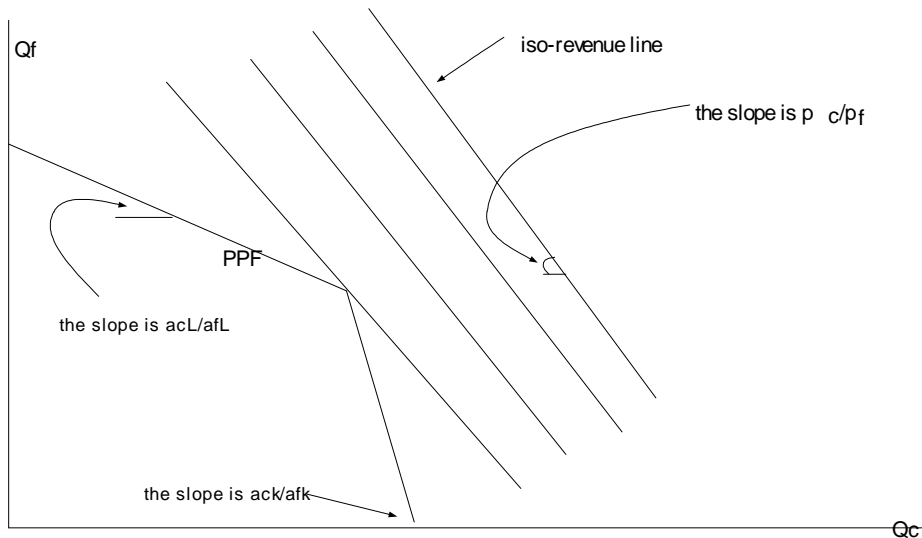
Let R be the revenue of this country. Then, the iso-revenue line is

$$R = p_c Q_c + p_f Q_f$$

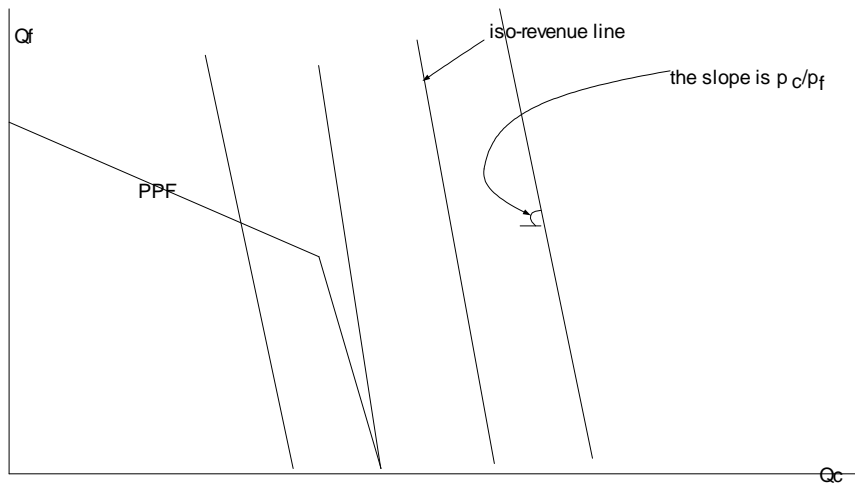
When we measure Q_f on the vertical axis and Q_c on the horizontal axis, the slope of the iso-revenue line is p_c/p_f

Thus, the optimal production for this country can be determined as follows:

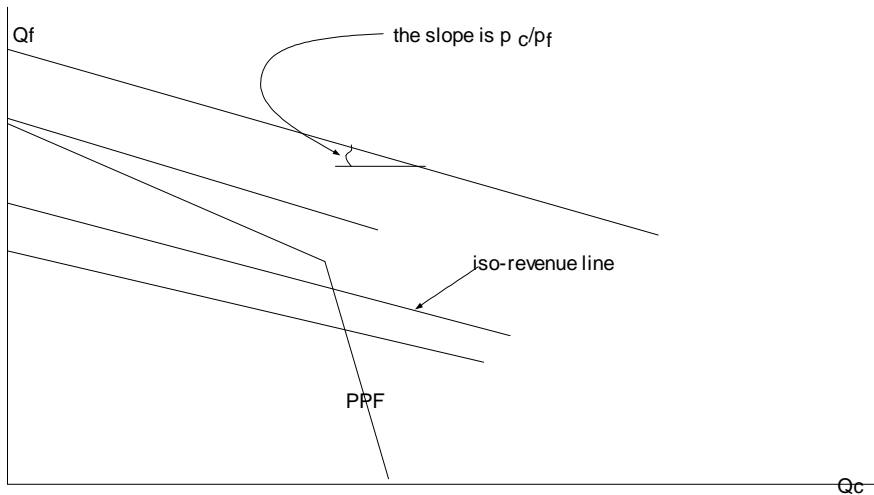
A case of incomplete specialization: both goods are produced



Complete specialization to clothing sector



complete specialization to food sector



Now first consider incomplete specialization case.

As it becomes clearer later, this assumption implies that this country's initial factor endowment is not so extreme.

Then, this country is likely to produce both goods at the equilibrium, which seem to be very reasonable

Incomplete specialization implies that p_c/p_f is greater than a_{cL}/a_{fL} and less than a_{cK}/a_{fK} .

$$\frac{a_{cL}}{a_{fL}} < \frac{p_c}{p_f} < \frac{a_{cK}}{a_{fK}}$$

Let r and w are the cost of using one unit of capital and labor respectively. In the clothing sector, to produce one unit of clothing, it costs $a_{cK}r + a_{cL}w$. This is the marginal and average cost in the clothing sector.

To produce one unit of food, it costs $a_{fK}r + a_{fL}w$

This is the marginal and the average cost in food sector.

If clothing is produced at the equilibrium, then under free entry and perfect competition, the following equation must hold

$$p_c = a_{cK}r + a_{cL}w$$

If food is produced at the equilibrium,

$$p_f = a_{fK}r + a_{fL}w$$

Assume that incomplete specialization

This implies that two goods are produced at the equilibrium.

Now we are ready to discuss four important theorems in Heckscher Ohlin model

Those theorems are

Rybczynski Theorem

Stolper-Samuelson Theorem

Factor price equalization Theorem

Heckscher-Ohlin Theorem

Rybczynski Theorem

The effect of a change of factor endowments in a small open economy

Assumptions

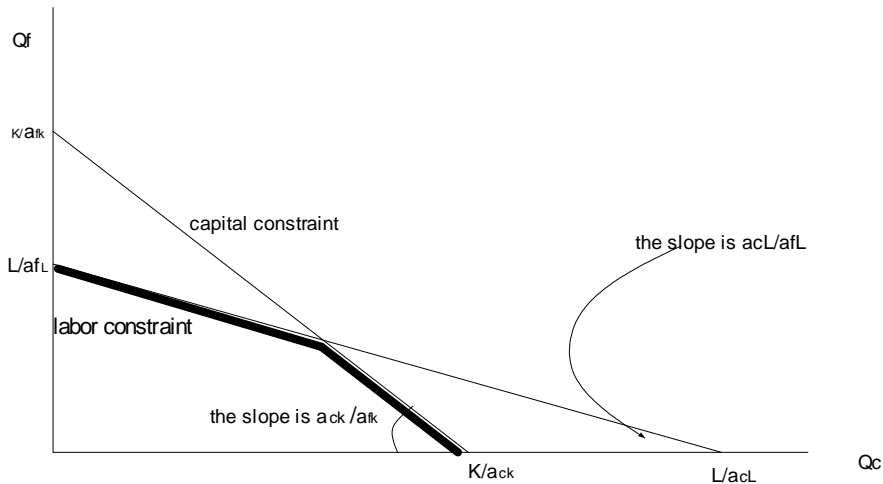
1. Both goods (clothing and food) are produced
2. International prices of clothing and food are given.

Rybczynski Theorem

Consider the Heckscher-Ohlin model. Assume that two goods are produced for given p_c and p_f . Suppose that one factor endowment increases and the price of p_c and p_f stay constant. . Then, a sector that is initially intensively using the factor that experience an increase of endowment will increase its production and the other sector will decrease its production

Proof of Rybczynski Theorem

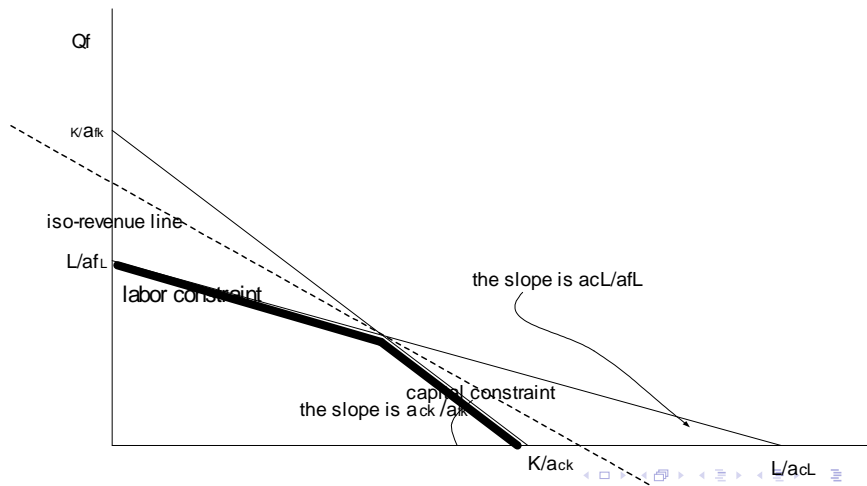
First draw the PPF.



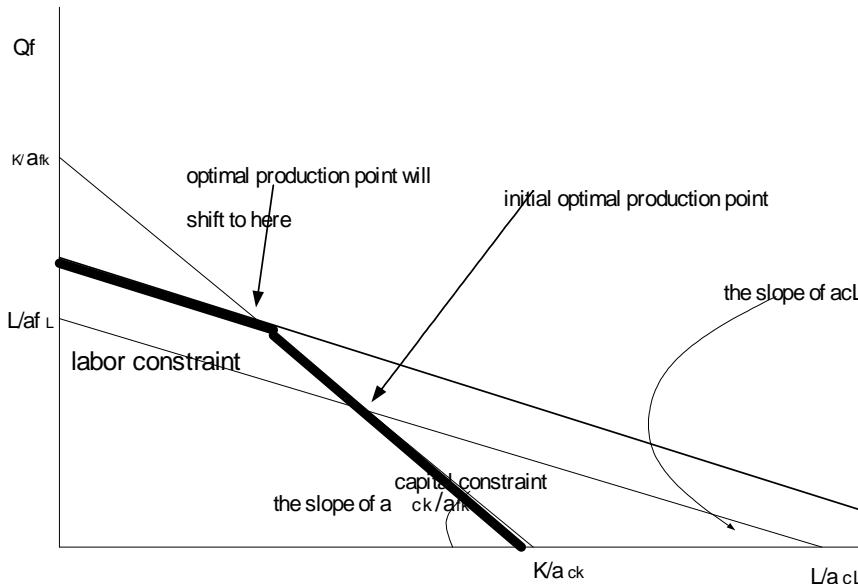
Since both goods are produced, we have

$$\frac{a_{cL}}{a_{fL}} < \frac{p_c}{p_f} < \frac{a_{cK}}{a_{fK}}$$

This implies that the iso-revenue line becomes as follows:



Now let's assume that all of sudden, the labor endowment increases due to acceptance of large number of immigrants
Then, labor constraint will shift outward. Thus, PPF changes as follows



From the above graph, it is clear that when L increases, Q_f increases and Q_c decreases.

In other words, when one factor endowment increases, the sector that is initially intensively using the factor that experience an increase of the endowment will increase its production. The other sector will decrease its production.

Similarly, you can analyze the case where K increases in this economy. In this case, Q_c increases and Q_f decreases.

Stolper-Samuelson Theorem

The effect of a change of a good price on the wage rate and rental rate of capital

Assumptions

1. A country is small and open.
2. p_c and p_f are given.
3. Two goods are produced before and after a change of the price of a good

This assumption implies that before and after a change of the price of a good, we have

$$\frac{a_{cL}}{a_{fL}} < \frac{p_c}{p_f} < \frac{a_{cK}}{a_{fK}}$$

Stolper-Samuelson Theorem

Consider the Heckscher-Ohlin model. Suppose that initially in this country both goods are produced for given p_c and p_f .

Now suppose that a price of one good increases and still two goods are produced. Then, the real price of the factor that is intensively used in a sector that experiences an increase of the output price will increase. The real price of the other factor will decrease.

Proof

When two goods are produced, it implies that

$$\frac{a_{cL}}{a_{fL}} < \frac{p_c}{p_f} < \frac{a_{cK}}{a_{fK}}$$

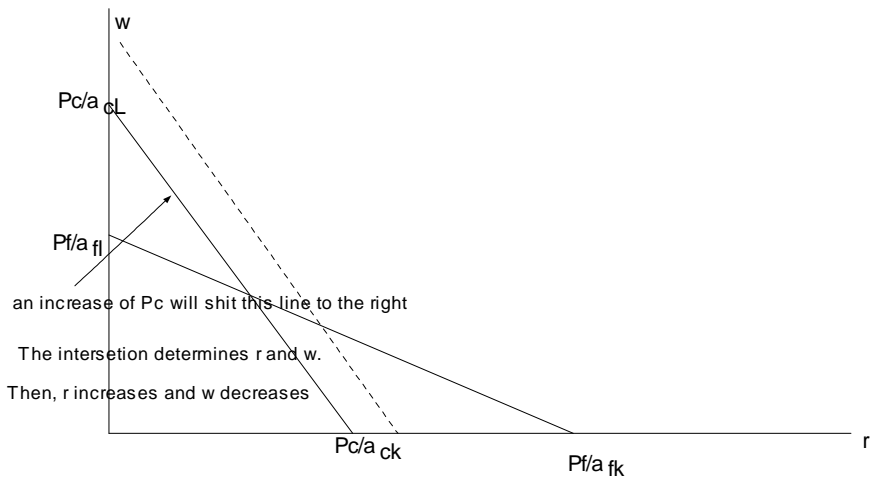
holds before and after a change of the price.

On the other hand, when two goods are produced, the average cost should be equal to the price in two sectors. Thus,

$$p_c = a_{cL}w + a_{cK}r$$

$$p_f = a_{fL}w + a_{fK}r$$

Now, let draw a graph of those two equation by measuring w on the vertical axis and r on the horizontal axis.



In equation, this can be explained as follows. Using the matrix, the above two equation can be written as

$$\begin{pmatrix} p_c \\ p_f \end{pmatrix} = \begin{pmatrix} a_{cL} & a_{cK} \\ a_{fL} & a_{fK} \end{pmatrix} \begin{pmatrix} w \\ r \end{pmatrix}$$

By taking the inverse the matrix, we have

$$\begin{pmatrix} a_{cL} & a_{cK} \\ a_{fL} & a_{fK} \end{pmatrix}^{-1} \begin{pmatrix} p_c \\ p_f \end{pmatrix} = \begin{pmatrix} w \\ r \end{pmatrix}$$

$$\frac{1}{D} \begin{pmatrix} a_{fk} & -a_{cK} \\ -a_{fL} & a_{cL} \end{pmatrix} \begin{pmatrix} p_c \\ p_f \end{pmatrix} = \begin{pmatrix} w \\ r \end{pmatrix}$$

where $D = a_{cL}a_{fK} - a_{cK}a_{fL}$

Note that clothing sector is capital intensive, we have

$$\frac{a_{cK}}{a_{cL}} > \frac{a_{fK}}{a_{fL}}$$

This implies that

$$\begin{aligned} a_{cK}a_{fL} &> a_{fK}a_{cL} \\ a_{cL}a_{fK} - a_{cK}a_{fL} &< 0 \\ D &< 0 \end{aligned}$$

Thus, we have

$$\begin{pmatrix} w \\ r \end{pmatrix} = \begin{pmatrix} \frac{a_{fk}}{D} p_c - \frac{a_{ck}}{D} p_f \\ -\frac{a_{fl}}{D} p_c + \frac{a_{cl}}{D} p_f \end{pmatrix}$$

Therefore, holding p_f constant, an increase of p_c will decrease w and increase r .

Now we consider the real factor price such as w/p_c , w/p_f , r/p_c , r/p_f
Since p_c increases, w/p_c decreases, w/p_f decreases, r/p_f increases. How
about r/p_c ? Note that

From the average cost is equal to the price, we have

$$p_c = a_{cK}r + a_{cL}w$$

Dividing by p_c on both sides, we have

$$1 = a_{cK} \frac{r}{p_c} + a_{cL} \frac{w}{p_c}$$

Since w/p_c decreases, r/p_c must increase.

Thus, w/p_c and w/p_f decrease and r/p_c and r/p_f increases.

Remark:

When p_c increases, the real factor price of capitalist increases either measured by p_c or p_f . The real wage of a worker measured either by p_c or p_f decreases. In other words, capitalist certainly becomes better off and worker certainly become worse off.

When p_f increases, the opposite will happen.

Factor Price Equalization Theorem

The effect of factor endowments on factor prices

Assumptions

1. N countries are engaged in free trade (no tariff, no non-tariff barrier)
2. Technologies of N countries are the same
3. In those N countries, both goods are produced
4. In those N countries, factor endowments are different

Factor Price Equalization Theorem

Consider N countries that are engaged in free trade. Assume that those N countries have the same technologies but the factor endowments are different. If those N countries produce both goods, each of factor prices are the same across countries.

Proof

For those N countries, the following two equation must hold as long as two goods are produced in those N countries

$$p_c = wa_{cL} + ra_{cK}$$

$$p_f = wa_{fL} + ra_{fK}$$

The above equations show that once p_c and p_f are given, then w and r are determined uniquely, independent of factor endowments.

Thus, as long as those N countries produce both goods, w are the same across N countries and r are the same in those N countries

Hechscher-Ohlin Theorem

The effect of factor endowments on trade pattern

Assumptions

1. Two countries
2. Same technologies in those two countries
3. Home is relatively more capital abundant
4. Foreign is relative more capital abundant

Assumption 3 and 4 implies that

$$\frac{K}{L} > \frac{K^*}{L^*}$$

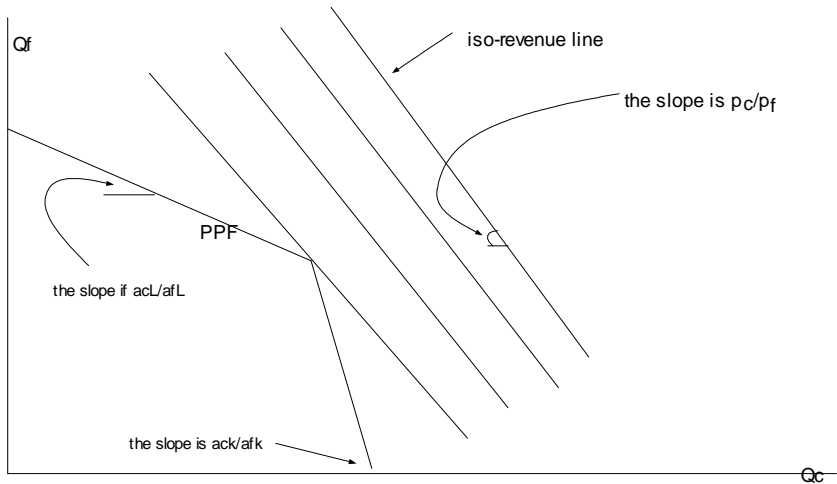
equivalently $\frac{L^*}{K^*} > \frac{L}{K}$

Heckscher-Ohlin Theorem

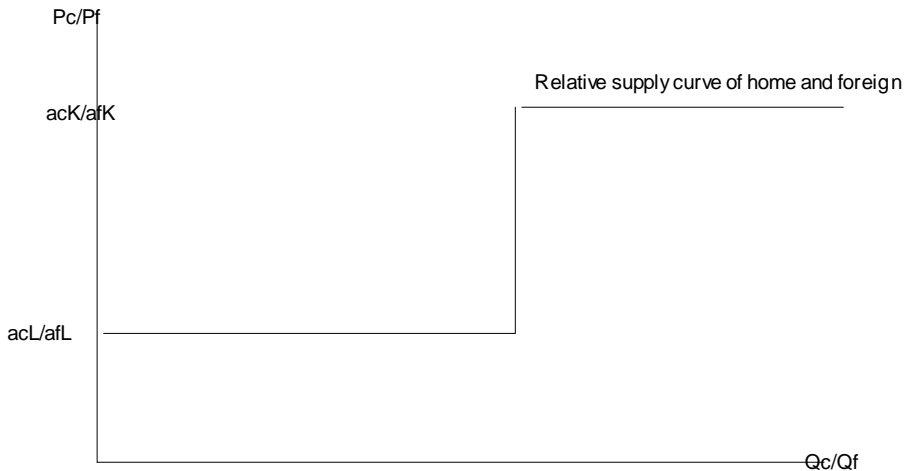
Consider now countries that have the same technologies. If two countries are engaged in free trade, then relatively capital abundant country will export capital intensive good. Relatively labor abundant county will export labor intensive good.

Proof

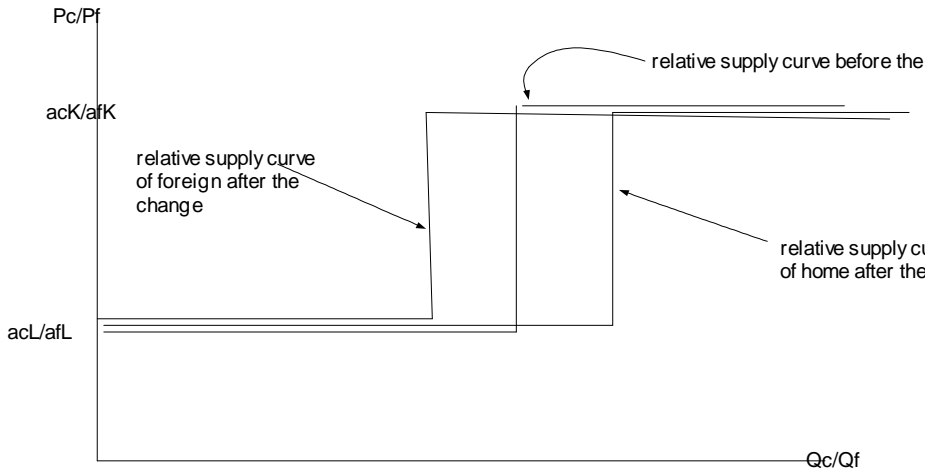
Consider the following thought experiment. Suppose that two countries are identical not only in technologies but also identical in terms of endowment. Thus, two countries are exactly the same. Then, to the relative supply curve of those two countries becomes the same. The relative supply curve can be written from the PPF and iso-revenue line:



Thus, the relative supply curve of those two countries become as follows:

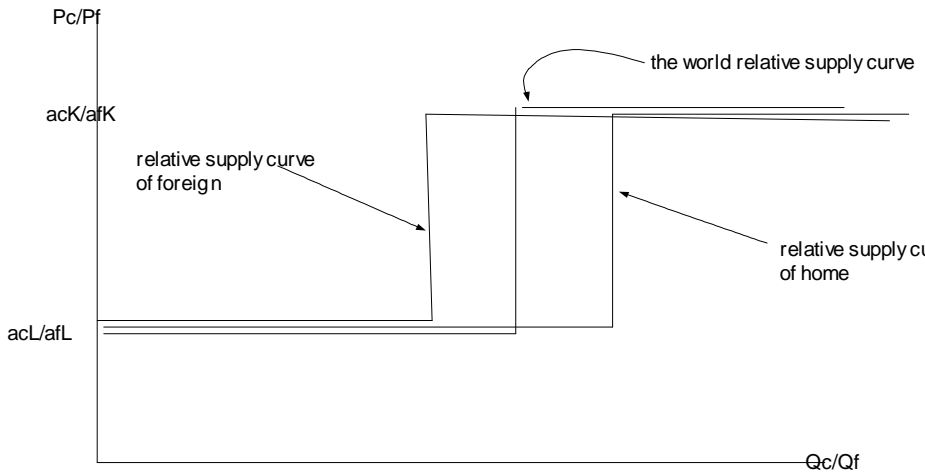


Now, continue our thought experiment. Assume that all of sudden, capital move from foreign countries to home county. As a result, home becomes relative capital abundant country and foreign becomes relative labor abundant country. Then, what will happen to the relative supply curve? The relative supply curve of home and foreign become as follows:

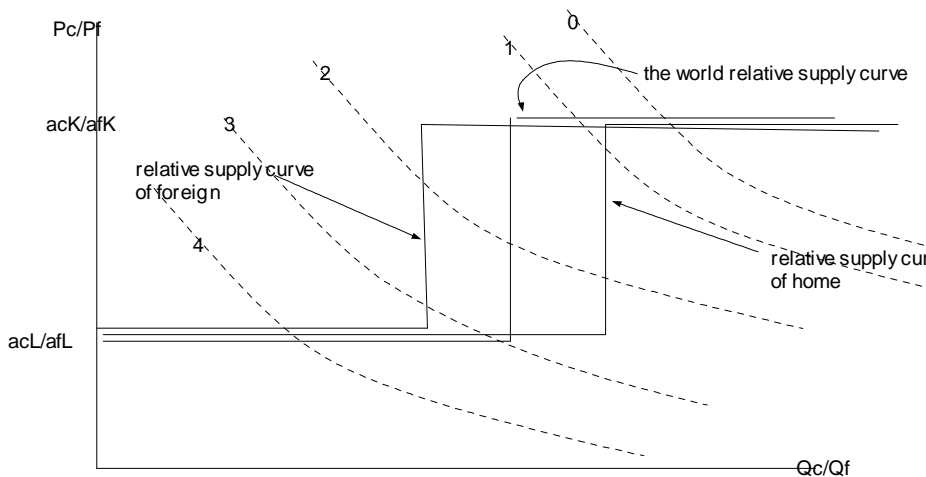


But why?

Let's continue our thought experiment.
Now assume that each country are different in endowment initially.
Assume that those two countries are initially closed. Then
then assume that all of sudden, two countries are engaged in free trade.
What will happen to those countries ?



Now consider the possible equilibrium pattern



The case 0 and case 4 are easy. No change of the relative price even after opening the border. No change of production and consumption. No trade at all after opening the border

In the case 1, in foreign p_c/p_f is equal to a_{cK}/a_{fK} when foreign is closed. For home, p_c/p_f is lower than a_{cK}/a_{fK} .

After opening the border, p_c/p_f will become a_{cK}/a_{fK} . Thus, p_c/p_f increases in home and will be the same in foreign.

In home, x_c/x_f will decrease. Q_c/Q_f can be the same or increase. Thus, home will export clothing and import food.

In foreign, p_c/p_f are the same. Thus, the relative demand, x_c^*/x_f^* is the same.

Thus, Q_c^* must go down and Q_f^* must increase. This is consistent with the equilibrium

Case 3 is the opposite of the case 1.

In the case 2, home will experience an increase of p_c/p_f . The foreign will experience a decrease of p_c/p_f .

As the result, the relative demand of clothing x_c/x_f will go down in home after trade.

In foreign, the relative demand of clothing x_c^*/x_f^* will increase.

Q_c/Q_f is the same even after opening the border

Q_c^*/Q_f^* is the same even after opening the border.

Thus, home must export clothing and import food.

Foreign must export food and import clothing

Summary of the Heckscher Ohlin Theorem

A country that is relatively abundant in capital will export capital intensive good and import labor intensive good. A country that is relative abundant in labor will export labor intensive good and import capital intensive good